Report on Foundations for Dynamic Equipment

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Report on Foundations for Dynamic Equipment

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This report presents to industry practitioners the various design criteria and methods and procedures of analysis, design, and construction applied to foundations for dynamic equipment.

Keywords: amplitude; foundation; reinforcement; vibration.

CONTENTS

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CHAPTER 1—INTRODUCTION

1.1—Background

Machinery with rotating, reciprocating, or impacting masses requires a foundation that can resist dynamic forces. Precise machine alignment should be maintained, and foundation vibrations should be controlled to ensure proper functioning of the machinery during its design service life.

Successful design of such foundations for dynamic equipment involves close collaboration and cooperation among machine manufacturers, geotechnical engineers, engineers, owners, and construction personnel. Because different manufacturers may have very different foundation acceptance criteria and their own practices with regards to foundation design requirements, strict adherence to ACI 318 alone may not be necessarily appropriate for certain foundations that support heavy industrial equipment, such as steam turbine generators, combustion turbine generators, or compressors. In addition, different practicing engineering firms may use design approaches based on past successful performance of foundations, even though these may not be the most economical designs. Therefore, this report summarizes current design practices to present a common approach, in principle, for various types of concrete foundations supporting dynamic equipment.

Compared to the previous edition, this document has been reorganized to make the document more systematic and user-friendly. More detailed information on the following subjects has been added on the behavior of foundations subjected to dynamic machine forces:

a) Impedance of the supporting medium (both soil-supported and pile-supported foundations)

b) General overview of vibration analysis (including finite-element modeling) and acceptance criteria, including finite-element analysis

c) Determination of various soil properties required for dynamic analysis of machine foundations

Example problems have been reworked and improved with some additional details to better illustrate the implementation of the calculation procedure in a manual calculation. Latest relevant references have been added to capture the current practice.

1.2—Purpose

The purpose of this report is to present general guidelines and current engineering practices in the analysis and design of reinforced concrete foundations supporting dynamic equipment.

This report presents and summarizes, with reference materials, various design criteria, methods and procedures of analysis, and construction practices currently applied to dynamic equipment foundations by industry practitioners.

1.3—Scope

This document is limited in scope to the engineering, construction, repair, and upgrade of concrete foundations for dynamic equipment. For the purposes of this document, dynamic equipment includes the following:

a) Rotating machinery
b) Reciprocating machinery
c) Impact or impulsive machinery

ACI 351.1R provides an overview of current design practice on grouting. Design practices for foundations supporting static equipment are discussed in ACI 351.2R.

There are many technical areas that are common to both dynamic equipment and static equipment foundations. Various aspects of the analysis design and construction of foundations for static equipment are addressed in ACI 351.2R. To simplify the presentation, this report is limited in scope to primarily address the design and material requirements that are pertinent only to dynamic equipment foundations. Engineers are advised to refer to ACI 351.2R for more information on the foundation design criteria (static loadings, load combinations, design strength, stiffness, and stability) and design methods for static loads. In particular, ACI 351.2R provides detailed coverage on the design of anchorage of equipment to concrete foundations. Note that

ACI 351.2R was published prior to a major revision to ACI 318 and some of the section numbers that it references in ACI 318 may have changed.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

A = steady-state vibration amplitude, in. (mm)
A_{head}, A_{crank} = head and crank areas, in.^{2} (mm^2)
A_p = cross-sectional area of the pile, in.^{2} (mm^2)
a, b = plan dimension of a rectangular foundation, ft (m)
a_o = dimensionless frequency
B_c = cylinder bore diameter, in. (mm)
B_i = mass ratio for the i-th direction
B_{mf} = machine footprint width, ft (m)
B_M = width of mat foundation, ft (m)
B_r = ram weight, tons (kN)
b_1, b_2 = constants 0.425 and 0.687, respectively
C = damping coefficient or total damping at center of resistance
$[C]$ = damping matrix
C_{CR} = critical damping coefficient
C_{i1},C_{i2} = dimensionless stiffness and damping parameters, subscription $i = u, v, \psi, \eta$
c = viscous damping constant, lbf-s/ft (N-s/m)
c_i = damping constant for the i-th direction
c_i(adj) = adjusted damping constant for the i-th direction
c_{ij} = equivalent viscous damping of pile j in the i-th direction
CG = center of gravity
CF = center of force
c_{gi} = pile group damping in the i-th direction
D = damping ratio
D_i = damping ratio for the i-th direction
D_{rod} = rod diameter, in. (mm)
d = pile diameter, in (mm)
d_s = displacement of the slide, in. (mm)
d_{mf} = distance from machine shaft centerline to top of foundation, ft (m)
E = static Young's modulus of concrete, psi (MPa)
E_d = dynamic Young's modulus of concrete, psi (MPa)
E_p = Young's modulus of the pile, psi (MPa)
e_m = mass eccentricity, in. (mm)
F = peak value of harmonic dynamic load (force or moment)
F_1 = correction factor
F_{block} = force acting outward on the block from which concrete stresses should be calculated, lbf (N)
$(F_{bolt})_{CHG}$ = force to be restrained by friction at the crosshead guide tie-down bolts, lbf (N)
$(F_{bolt})_{frame}$ = force to be restrained by friction at the frame tie-down bolts, lbf (N)
F_D = damper force, lbf (N)
F_{GMAX} = maximum horizontal gas force on a throw or cylinder, lbf (N)
F_{IMAX} = maximum horizontal inertia force on a throw or cylinder, lbf (N)
F_K = force in vibration isolator spring, lbf (N)
F_o = dynamic force amplitude (zero-to-peak), lbf (N)
F_{pl} = lateral/longitudinal pseudo-dynamic design force, lbf (N)
F_{pv} = vertical pseudo-dynamic design force, lbf (N)
F_r = maximum horizontal dynamic force, lbf (N)
F_{red} = force reduction factor to account for the fraction of individual cylinder load carried by the compressor frame (frame rigidity factor)
F_{rod} = force acting on piston rod, lbf (N)
F_s = dynamic inertia force of slide, lbf (N)
F_{THROW} = horizontal force to be resisted by each throw's anchor bolts, lbf (N)
$F(t)$ = generic representation of time-varying load (force or moment) horizontal
$F_{unbalance}$ = maximum value applied using parameters for a horizontal compressor cylinder, lbf (N)
f_c' = specified concrete compressive strength, psi (MPa)
f_{i1}, f_{i2} = dimensionless pile stiffness and damping functions for the i-th direction
f_o = operating speed, rpm
G, G^* = dynamic shear modulus of the soil, psi (MPa)
G_pJ = torsional stiffness of the pile, lbf-ft^2 (N-m^2)
G_s = dynamic shear modulus of the embedment (side) material, psi (MPa)
H = depth of soil layer, ft (m)
I_g = gross area moment of inertia, in.^{2} (mm^2)
I_p = moment of inertia of the pile cross section in.^{4} (mm^4)
i = $\sqrt{-1}$
i = directional indicator or modal indicator, as a subscript
K = stiffness or total stiffness at center of resistance, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
$[K]$ = stiffness matrix
K' = total stiffness at center of gravity, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
K_{ij}^* = impedance in the i-th direction due to a displacement in the j-th direction
K_N = actual negative stiffness, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
K_P = arbitrary chosen positive stiffness value (typically set equal to the static stiffness), lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
K_{eff} = effective bearing stiffness, lbf/in. (N/mm)
K_s = static soil stiffness, lbf/in^3 (N/m^3)
K_c^G = pile group coupling impedance
K_h^G = pile group horizontal impedance
K_v^G = pile group vertical impedance
K_ψ^G = pile group rocking impedance
k = individual pile stiffness at center of resistance, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
k_{ei}^* = impedance in the i-th direction due to embedment
k_{gi} = pile group stiffness in the i-th direction, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
k_i = static stiffness for the i-th direction, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)

k_i^* = frequency-dependent impedance in the i-th direction
k_i(adj) = adjusted static stiffness for the i-th direction, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
k_i^*(adj)= adjusted frequency-dependent impedance in the i-th direction
k_{ij} = stiffness of pile j in the i-th direction, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
k_{ij}^{st} = static stiffness of an individual pile j in the i-th direction, lbf/ft (N/m) or lbf-ft/rad (N-m/rad)
k_s = soil modulus of subgrade reaction, lbf/in^3 (N/m^3)
k_{st} = static stiffness constant
k_u^* = horizontal impedance of supporting medium
k_v^* = vertical impedance of supporting medium
k_ψ^* = rocking impedance of supporting medium
k_η^* = torsional impedance of supporting medium
$k(\omega)$= frequency (ω)-dependent dynamic impedance
L = length of connecting rod, in. (mm)
L_M = greater plan dimension of the mat foundation, ft (m)
L_{mf} = machine footprint length, ft (m)
L_P = lateral distance from center of resistance to individual piles, ft (m)
l = depth of embedment, ft (m)
l_p = pile length, ft (m)
M = mass, lbm (kg)
$[M]$ = mass matrix
M_h = hammer mass, including any auxiliary foundation, lbm (kg)
M_o = overturning moment on foundation, lbf-ft (N-m)
MR = mass ratio of concrete foundation to machine
M_r = ram mass, including dies and ancillary parts, lbm (kg)
M_{res} = foundation overturning resistance, lbf-ft (N-m)
M_Δ = added mass, lbm (kg)
m = mass of the machine-foundation system; lbm (kg)
m_d = slide mass including the effects of any balance mechanism, lbm (kg)
m_r = rotating mass, lbm (kg)
m_{rec} = reciprocating mass in a reciprocating machine, lbm (kg)
m_{rot} = rotating mass in a reciprocating machine, lbm (kg)
m_s = added mass (inertial), lbm (kg)
N = number of piles
$(N_{bolt})_{CHG}$ = number of bolts holding down one crosshead guide
$(N_{bolt})_{frame}$ = number of bolts holding down the frame, per cylinder
NT = normal torque, lbf-ft (N-m)
P_{ALL} = allowable bearing pressure, ksf (kPa)
P_{head},
P_{crank}= instantaneous head and crank pressures, psi (MPa)
P_{max}= maximum bearing pressure, ksf (kPa)
P_s = power being transmitted by the shaft at the connection, horsepower (kilowatts)
R = circular foundation radius, equivalent translation radius of rectangular foundation, ft (m)
R_i = equivalent radius of rectangular foundation, ft (m)
$R_{\psi a}$, $R_{\psi b}$ = equivalent rocking radius of foundation about a- and b-axis, respectively, ft (m)
R_η = equivalent torsional radius of foundation, ft (m)
r = length of crank, in. (mm)
r_i = radius of the crank mechanism of the i-th cylinder, in. (mm)
r_o = pile radius or equivalent radius, in. (mm)
S = press stroke, in. (mm)
S_{all} = allowable foundation settlement, in. (mm)
S_f = service factor, used to account for increasing unbalance during the design service life of the machine
S_{i1}, S_{i2} = dimensionless stiffness and damping parameters for side layer, subscription $i = u, v, \psi, \eta$
S_{max} = maximum foundation settlement, in. (mm)
SV_R = seismic shear force due to the rigid foundation and other rigid components, lbf (N)
SVs = seismic shear force due to the superstructure, machine and other flexible components, lbf (N)
$SV_{seismic}$=total seismic shear force machine-foundation system, lbf (N)
s = pile center-to-center spacing, ft (m)
$[T]$ = transfer matrix
T_M = mat foundation thickness, ft (m)
T_{min} = minimum required anchor bolt tension, lbf (N)
t = time, s
u = displacement amplitude, in. (mm)
u_0 = peak displacement amplitude, in. (mm)
V_c = compressive velocity of a pile, ft/s (m/s)
V_F = transmissibility factor
V_{La} = Lysmer's analog wave velocity, ft/second (m/s)
V_{max}= maximum allowable bearing vibration, in. (mm)
V_{peak}= peak velocity, in./s (mm/s)
V_{RMS}= root mean square velocity, in./s (mm/s)
V_s = shear wave velocity of the soil, ft/s (m/s)
v_h = post-impact hammer velocity, in./s (mm/s)
v_o = reference velocity = 18.4 ft/s (5.6 m/s) from a free fall of 5.25 ft (1.6 m)
v_r = ram impact velocity, ft/s (m/s)
W_a = equipment weight at anchorage location, lbf (N)
W_f = weight of the foundation, tons (kN)
W_m = machine weight, tons (kN)
W_r = rotating weight, lbf (N)
y = generic representation of displacement (translational or rotational), in. (mm) or rad
y' ($\dot{y}$)= generic representation of velocity (translational or rotational), in./s (mm/s) or rad/s
y'' ($\ddot{y}$)= generic representation of acceleration (translational or rotational), in./s^2 (mm/s^2) or rad/s^2
y_c = crank pin displacement in local y-axis, or distance from the center of gravity to the base support, in. (mm)
y_e = distance from the center of gravity to the level of embedment resistance, ft (m)
z_c = crank pin displacement in local z-axis, in. (mm)
z_p = piston displacement, in. (mm)
α = angle between battered piles and vertical piles, rad
α_h = ram rebound velocity to impact velocity ratio

α_i = pile dynamic interaction factor in the i-th direction, subscription $i = z$ (axial), HH (horizontal), MM (in phase rocking), MH (sway rocking)
α_j = coefficients, $j = 1$
β_i = rectangular footing coefficient for the i-th direction
β_j, γ_j = coefficients, $j = 1$ to 4
β_m = material damping ratio
γ_c = concrete density, lbf/ft^3 (kN/m^3)
ϕ = phase angle
$\{\phi\}$ = mode shape vector
ϕ_i = mode shape factor
η = tuning ratio
θ = phase angle, or angle between the direction of load action and the plane in which piles lie, rad
ρ = soil mass density, lbm/ft^3 (kg/m^3)
ρ_p = pile mass density, lbm/ft^3 (kg/m^3)
Δ = peak amplitude (translational or rotational), in. or rad
μ = coefficient of friction
ν = Poisson's ratio of the soil
ω = circular frequency of motion, rad/s
ω_d = damped circular natural frequency, rad/s
ω_i = undamped circular natural frequency for the i-th mode, rad/s
ω_n = undamped circular natural frequency, rad/s
ω_o = circular operating frequency of a machine or other driving force, rad/s
ω_{su}, ω_{sv} = circular natural frequencies of a soil layer in horizontal (u) and vertical (v) directions, rad/s
u, v,
ψ, η = subscriptions used for notating horizontal, vertical, rocking, and torsional direction, respectively

2.2—Definitions

Please refer to the latest version of ACI Concrete Terminology for a comprehensive list of definitions. Definitions provided herein complement that resource.

root cause analysis—collective term that describes a wide range of approaches, tools, and techniques used to uncover causes of problems.

CHAPTER 3—FOUNDATION AND MACHINE TYPES

3.1—General considerations

The type, configuration, and installation of a foundation or support structure for dynamic machinery may depend on the following factors:

a) Site conditions such as soil characteristics, topography, seismicity, climate, and other effects

b) Machine base configuration such as frame size, cylinder supports, pulsation bottles, drive mechanisms, and exhaust ducts

c) Process requirements such as elevation requirements with respect to connected process equipment and support requirements for piping

d) Anticipated loads such as the equipment static weight, along with loads developed during construction, startup, operation, shutdown, and maintenance

e) Allowable amplitudes of vibration associated with each dynamic load case

f) Construction requirements such as limitations or constraints imposed by construction equipment, procedures, techniques, or the sequence of construction

g) Operational requirements such as accessibility, settlement limitations, temperature effects, and drainage

h) Maintenance requirements such as temporary access, laydown space, in-plant crane capabilities, and machine removal considerations

i) Regulatory factors, owner requirements, or building code provisions such as tied pile caps in seismic zones

j) Economic factors such as capital cost, useful or design service life, and replacement or repair cost

k) Environmental requirements such as secondary containment or special concrete coating requirements

l) Recognition that certain machines, particularly large reciprocating compressors, rely on the foundation to add strength and stiffness that is not inherent in the structure of the machine

3.2—Machine types

3.2.1 *Rotating machinery*—This category includes gas turbines, steam turbines, and other expanders; turbo-pumps and compressors; fans; motors; and centrifuges. These machines are characterized by the motion of rotating components.

Unbalanced forces in rotating machines are created when the mass centroid of the rotating component does not coincide with the center of rotation (Fig. 3.2.1). This dynamic force is a function of the mass of the rotating component, speed of rotation, and the magnitude of the eccentricity of offset. The offset or eccentricity should be minor under manufactured conditions when the machine is well balanced, clean, and without wear or erosion. Changes in alignment, operation near resonance, turbine blade loss, and other malfunctions or undesirable conditions can greatly increase the force applied to its bearings by the rotor.

3.2.2 *Reciprocating machinery*—For reciprocating machinery, such as compressors or diesel engines, a piston moving in a cylinder interacts with a gas through the kinematics of a slider crank mechanism driven by, or driving, a rotating crankshaft. Individual inertia forces from each cylinder are inherently unbalanced with dominant frequencies at one and two times the rotational frequency (Fig. 3.2.2).

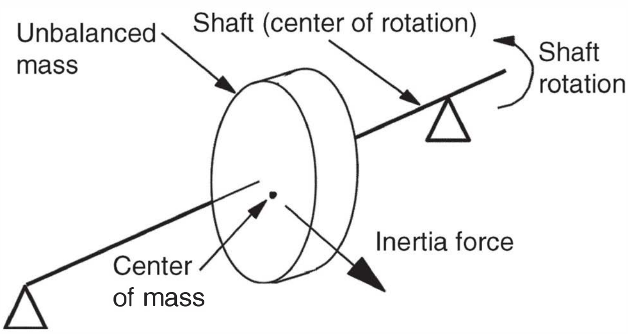


Fig. 3.2.1—Rotating machine diagram.

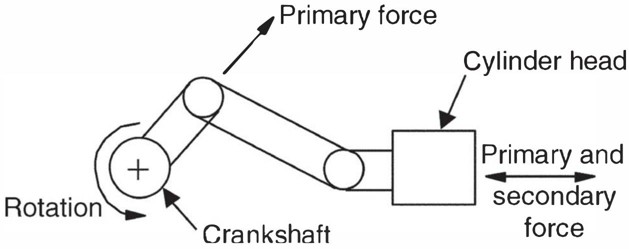


Fig. 3.2.2—Reciprocating machine diagram.

The unbalanced forces and moments generated by reciprocating machines with more than one piston are dependent on the crank arrangement. The optimum crank arrangement that minimizes loading is generally not possible because the mechanical design will be optimized to satisfy the operating requirements. This leads to piston/cylinder assemblies and crank arrangements that do not completely counter-oppose; therefore, unbalanced loads occur, which should be resisted by the foundation.

Individual cylinder fluid forces act outward on the cylinder head and inward on the crankshaft (Fig. 3.2.2). For a rigid cylinder and frame, these forces are internally balanced in the machine, but deformations of large machines can cause a significant portion of the forces to be transmitted to the mounts and into the foundation. Particularly on large reciprocating compressors with horizontal cylinders, it is inappropriate and unconservative to assume the compressor frame and cylinder are sufficiently stiff to internally balance all forces. Such an assumption has led to many inadequate mounts for reciprocating machines.

3.2.3 *Impulsive machinery*—Equipment, such as forging hammers and some metal-forming presses, operate with regulated impacts or shocks between different parts of the equipment. This shock loading is often transmitted to the foundation system of the equipment and can propagate into the surroundings and is a factor in the design of the foundation.

Closed die forging hammers typically operate by dropping a weight (ram) onto hot metal, forcing it into a predefined shape. While the intent is to use this impact energy to form and shape the material, there is significant energy transmission, particularly late in the forming process. During these final blows, the material being forged is cooling and less shaping takes place. Thus, pre-impact kinetic energy of the ram converts to post-impact kinetic energy of the entire forging hammer. As the entire hammer moves downward, it becomes a simple dynamic mass oscillating on its supporting medium. This system should be well damped so that the oscillations decay sufficiently before the next blow. Timing of the blows commonly range from 40 to 100 blows per minute. The ram weights vary from a few hundred pounds to 35,000 pounds (16 tons). Impact velocities in the range of 25 ft/s (7.6 m/s) are common. Open die hammers operate in a similar fashion but are often of two-piece construction with a separate hammer frame and anvil.

Forging presses perform a similar manufacturing function as forging hammers but are commonly mechanically or hydraulically driven. These presses form the material at low velocities but with greater forces. The mechanical drive system generates horizontal dynamic forces that the engineer should consider in the design of the support system. Rocking stability of this construction is important. Figure 3.2.3 shows a typical example of a horizontal forcing function through one full stroke of a forging press.

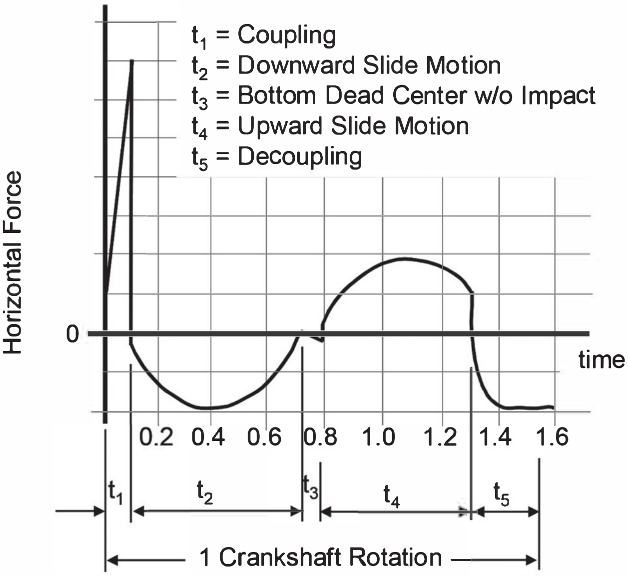


Fig. 3.2.3—Example of a forcing function for a forging press.

Mechanical metal forming presses operate by squeezing and shearing metal between two dies. Because this equipment can vary greatly in size, weight, speed, and operation, forces and design criteria used for the foundation design can vary greatly. Speeds can vary from 30 to 1800 strokes per minute. Dynamic forces from the press develop from two sources: the mechanical imbalance of the moving parts in the equipment and the response of the press frame as the material is sheared (snap-through forces). Imbalances in the mechanics of the equipment can occur both horizontally and vertically. Generally, high-speed equipment is well balanced. Low-speed equipment is often not balanced because the inertia forces at low speeds are small. The dynamic forces generated by all of these presses can be significant as they are transmitted into the foundation and propagate into the subgrade.

3.2.4 *Other machine types*—Other machinery generating dynamic loads include rock crushers and metal shredders. While part of the dynamic load from these types of equipment tend to be based on rotating imbalances, there are also random characteristics to the dynamic forces that vary with the particular operation and design.

3.3—Foundation types

3.3.1 *Block-type foundation*—Dynamic machines are preferably located close to grade to minimize the elevation difference between the machine dynamic forces and the center of gravity of the machine-foundation system (Fig. 3.3.1). The low location also reduces the moments due to horizontal forces. The ability to use such a founda-

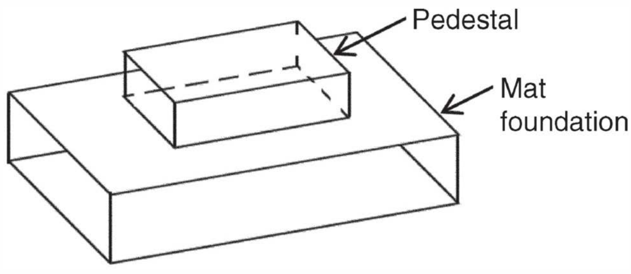


Fig. 3.3.1—Block-type foundation.

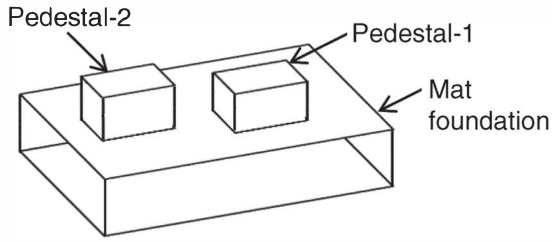


Fig. 3.3.2—Combined block-type foundation.

tion primarily depends on the quality of surface soils. Block foundations are nearly always designed as rigid structures. The dynamic response of a rigid block foundation depends only on the dynamic load, mass, dimensions, and soil characteristics.

3.3.2 *Combined block-type foundation*—Combined blocks are used to support closely spaced machines (Fig. 3.3.2). When designing combined block-type foundations, it is important to consider the combination of forces from two or more machines and possible lack of stiffness of a larger mat foundation.

3.3.3 *Tabletop-type foundation*—Elevated support is needed when access is required to the underside of the machinery for ducts, piping, maintenance platforms, or the machine has to be elevated for process reasons (Fig. 3.3.3). Tabletop structures are considered to be flexible, hence their response to dynamic loads can be quite complex and depend both on the motion of its discrete structural members (columns, beams, and footing) and the subgrade upon which it is supported.

3.3.4 *Tabletop with isolators*—Isolators (springs and dampers) located at the top of supporting columns are sometimes used to minimize the response to dynamic loading (Fig. 3.3.4). The effectiveness of isolators (or the degree of isolation, as defined in Chapter 6) depends on the damping, machine speed, and natural frequency of the foundation. Descriptions for the technical details of this type of support are provided in 6.3.3.

3.3.5 *Spring-mounted equipment on block foundation*—Occasionally, machinery are mounted on springs to minimize forces from connecting piping (Fig. 3.3.5). The springs are then supported on a block-type foundation. This arrangement has a dynamic effect similar to that for tabletops with vibration isolators. Other types of equipment are spring mounted to limit the transmission of dynamic forces. Technical details are described in 6.3.3.

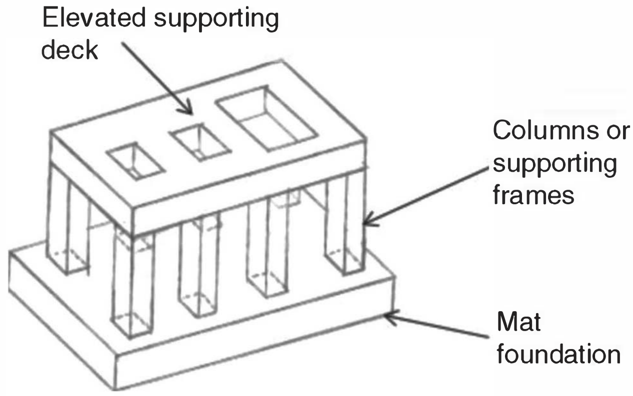


Fig. 3.3.3—Tabletop-type foundation.

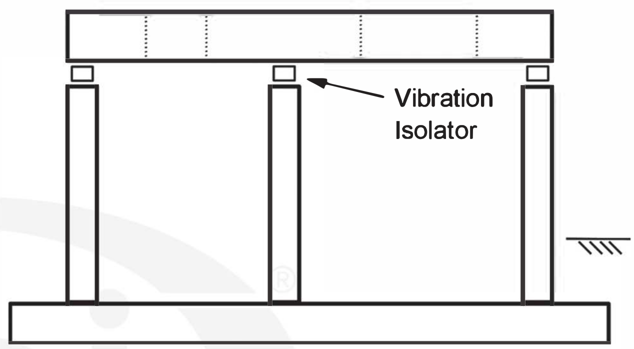


Fig. 3.3.4—Tabletop with isolators.

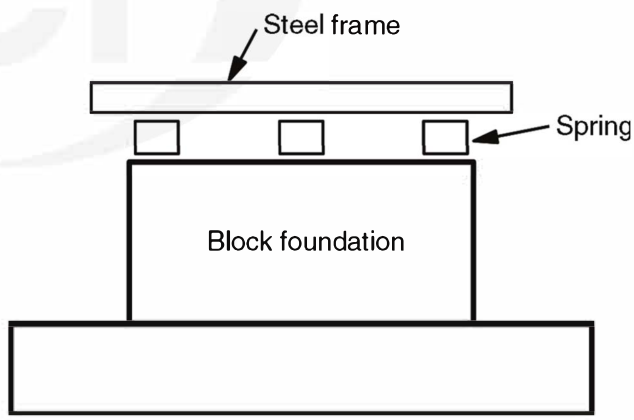


Fig. 3.3.5—Spring-mounted equipment on block foundation.

3.3.6 *Inertia block*—Dynamic equipment on a structure may be relatively small in comparison to the overall size of the structure. In this situation, dynamic machines are usually designed with a supporting inertia block to shift structure/machine natural frequencies away from machine operating speeds and reduce amplitudes by increasing the inertia (Fig. 3.3.6).

3.3.7 *Deep foundations*—Any of the previously mentioned foundation types may be supported directly on soil or on deep foundations such as piles or caissons (Fig. 3.3.7). Deep foundations are generally used where soft ground conditions result in low allowable bearing pressures and excessive

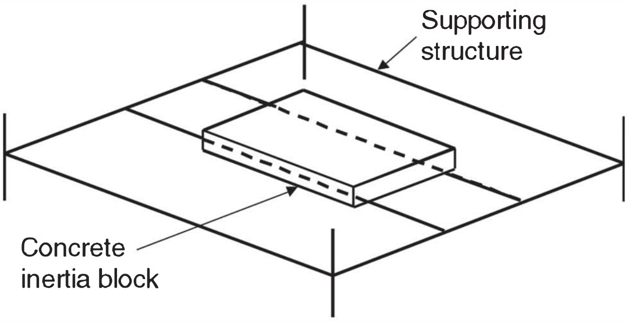


Fig. 3.3.6—Inertia block.

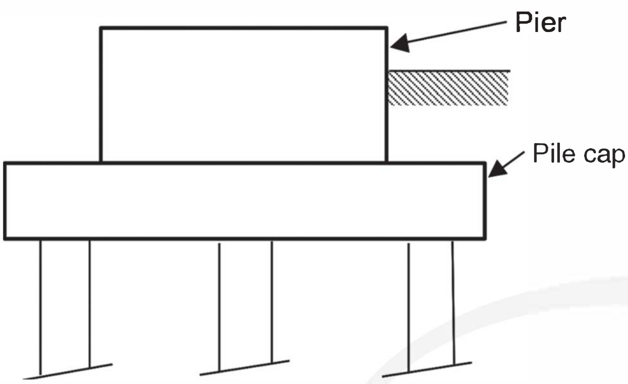


Fig. 3.3.7—Deep foundation.

settlement for a mat-type foundation. Piles use end bearing, frictional side adhesion, or a combination of both, to transfer axial loads into the underlying soil. Transverse loads are resisted by soil pressure bearing against the side of the pile cap and piles or by battered piles. Various types of piles are used, including drilled piers, caissons, auger cast piles, and driven piles.

CHAPTER 4—DESIGN LOADS

4.1—Overview of design loads and criteria

The design of concrete foundations that support dynamic machinery involves several tasks:

a) Defining the anticipated loads

b) Properly transferring these loads to the foundation and then to the supporting medium (supporting soil, pile, or structure)

c) Establishing the performance criteria, and providing for these through proper proportioning and detailing of structural members.

Behind this straightforward series of tasks lies the need for careful attention to the interfaces between machine, mounting system, concrete foundation, and the interaction with subgrade media.

The loads on machine foundations may be divided into one of three categories: equipment loads; environmental loads; and installation/maintenance loads.

The equipment loads on machine foundations may be both static and dynamic. Static loads are principally a function of the weights of the machine and all its auxiliary equipment. Dynamic loads, which occur during the operation of the machine, result from forces generated by unbalance, inertia of moving parts, or both, and by the flow of fluids and gases for some machines. The magnitude of dynamic loads varies as a function of time; and primarily depends upon the machine's operating speed and the type, size, weight, and arrangement (position) of moving parts within the machine casing.

The basic goal in the design of a machine foundation is to limit the amplitude of motion for the machine and foundation. The goal is to neither endanger the satisfactory operation of the machine nor disturb people working in the immediate vicinity (Gazetas 1983). Allowable amplitudes depend on the speed, location, and criticality or function of the machine. Other limiting dynamic criteria affecting the design may include avoiding resonance and excessive transmissibility to the supporting soil or structure. Thus, a key ingredient to a successful design is the careful engineering analysis of the soil-foundation response to dynamic loads from the machine operation.

The foundation's response to dynamic loads can be significantly influenced by the supporting medium (soil or subgrade/bedrock) on which it is constructed. Consequently, critical soil parameters, such as the dynamic soil shear modulus, are preferably determined from a field investigation and laboratory tests rather than relying on generalized correlations based on broad soil classifications. Due to the inherent variability of soil, the dynamic response of machine foundations is often evaluated using a range of values for the critical soil properties (as discussed in Chapter 5 and Appendix A).

Furthermore, a machinery support structure or foundation is designed with adequate structural strength to resist the worst possible combination of loads occurring over its design service life. This often includes limiting soil-bearing pressures to well within allowable limits to ensure a more predictable dynamic response and prevent excessive settlements and soil failures.

Foundations supporting reciprocating or rotating compressors, turbines, generators and motors, presses, and other machinery should withstand all the forces that may be imposed on them during their design service life. Machine foundations are unique because they may be subjected to significant dynamic loads during operation in addition to gravity, wind, and seismic loads. The magnitude and characteristics of the operating loads depend on the type, size, speed, and layout of the machine.

Generally, the weight of the machine, center of gravity, surface area(s), and operating speeds are readily available from the manufacturer of the machine. Establishing appropriate values for dynamic loads is best accomplished through careful communication and clear understanding between the machine manufacturer, owner, and engineer responsible for the foundation design. The communication should include but not be limited to the purpose, planned use for the loading information, and the definition of the information provided. It is in the best interest of all parties (machine manufacturer, engineer responsible for the foundation design, installer, owner, and operator) to ensure effective definition and communication of data and its appropriate use. Machines always experience some level of unbalance, vibration, and force transmitted through the bearings. Under some off-design conditions, such as wear, the forces may increase significantly. The machine manufacturer and engineer responsible for the foundation design should work together so that their combined knowledge achieves an integrated machine-foundation system that robustly serves the needs of its owner and operator and withstands design loads.

Usually the machine loads, including normal operation loads, emergency loads, catastrophic loads, and accidental loads, are provided by the machine manufacturer. Seismic loads and their distribution pattern from machine to the foundation at the machine support points should be provided by the manufacturer based on the machine configuration and mounting system. The total seismic loads for the foundation analysis should include the seismic loads induced from the machine mass and the seismic loads induced from foundation concrete mass. However, the engineer responsible for the foundation design should have a good understanding of each load and how the seismic load is transferred to the foundation. The machine seismic loads provided by the machine vendor may need to be adjusted based on the seismic design code requirements. Seismic loads from the mat foundation are, in general, out of phase from the seismic loads due to the machine and superstructure, unless the machine or superstructure are rigid. Therefore, the seismic shear due to the foundation and other rigid members (SV_R) and the seismic shear due to the superstructure + machine and other flexible (SV_S) members should be calculated by similar combinations such as square root of sum of the squares (SRSS) approach as follows

$$SV_{seismic} = (SV_R^2 + SV_S^2)^{0.5} \tag{4.1}$$

Close cooperation and coordination with the machine vendor is essential for a successful foundation design.

Sections 4.2 to 4.4 provide some of the commonly used loading descriptions, evaluations, or both, for determining machine-induced forces and other design loads for foundations supporting machinery. They include definitions and other information on static and dynamic loads. These loads should be provided by the machine manufacturer, and alternative assumptions should be made when such data are unavailable or are under-predicted.

4.2—Static machine loads

4.2.1 *Dead loads*—A major function of the foundation is to support gravity (dead) loads due to the weight of the machine, auxiliary equipment, piping, valves, and dead weight of the foundation structure. The weights of the machine components are normally supplied by the machine manufacturer. The distribution of the weight of the machine on the foundation depends on the location of support points and on the rigidity or flexibility of the machine frame. Typically, there are multiple support points and, thus, the distribution is statically indeterminate. In many cases, the machine manufacturer provides a loading diagram showing the vertical loads at each support point. Sometimes the center of gravity location of the machine components provided by the machine manufacturer can be used to distribute machine dead loads to the supporting points.

Some machine manufacturers provide the weights of turbine and generator at certain support points. These forces are due to axial movements of the turbine-generator rotor only at the bearing casing connection points of the combined radial/axial bearing (thrust bearing). These weights should be used in the dynamic analysis, seismic analysis, or both, of the foundation.

4.2.2 *Live loads*—Live loads are produced by personnel, tools, and maintenance equipment and materials. The live loads used in design should be the maximum loads expected during the design service life of the machine. For most designs, live loads are uniformly distributed over the floor areas of platforms of elevated support structures or to the access areas around at-grade foundations. Typical live loads vary from 60 lbf/ft^2 (3 kPa) for personnel to as much as 250 lbf/ft^2 (12 kPa) for maintenance equipment and materials (ASCE/SEI 7). For steam turbine generator machines, it is common to also consider a value of 400 to 500 lbf/ft^2 (19 to 24 kPa) for machine parts (such as rotors and casings) in a laydown area.

4.2.3 *Normal operating loads*—Static operating loads include the weight of the machine contents during normal operation. Static operating loads include forces such as the drive torque developed by some machines at the connection between the drive mechanism and driven machinery. Static operating loads can also include forces caused by thermal growth of the machinery equipment and connecting piping. Time-varying (dynamic) loads generated by machines during operation are covered in 4.3.

4.2.3.1 *Operating torque loads*—Machines such as compressors, turbines, and generators require some form of drive mechanism, either integral with the machine or separate from it. When the drive mechanism is nonintegral, such as a separate electric motor, reciprocating engine, and gas or steam turbine, it produces a net external drive torque on the driven machine. For turbine-generators, fluid or magnetic coupling between the rotor and stator of the generator can produce an overturning torque about the longitudinal axis (shaft) equal in magnitude and opposite in direction on the driver and driven machine. For example, the normal operational torque load from turbines is oriented in the opposite direction to that of shaft rotation, whereas for the generator, the normal operational torque load is oriented in the direction of shaft rotation (Fig. 4.2.3.1a). The normal torque (sometimes called drive torque) is generally applied to the foundation as a static force couple in the vertical direction acting about the centerline of the shaft of the machine based on the highest expected output of the machine. The magnitude of the normal torque is often calculated as

$$NT = \frac{(5250)(P_s)}{f_o} \text{ (lbf-ft)}$$
$$NT = \frac{(9550)(P_s)}{f_o} \text{ (N-m)} \tag{4.2.3.1a}$$

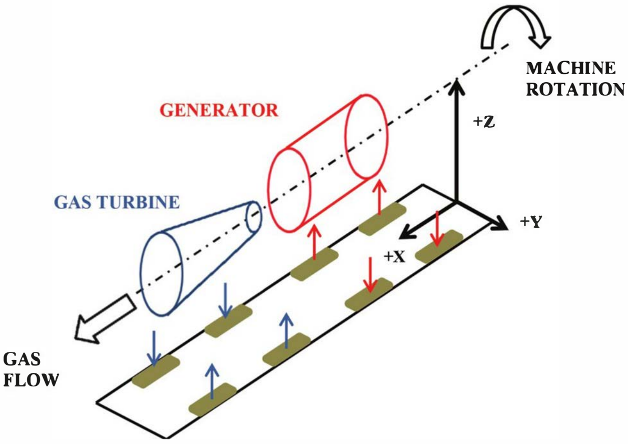


Fig. 4.2.3.1a—Equivalent forces for torque loads.

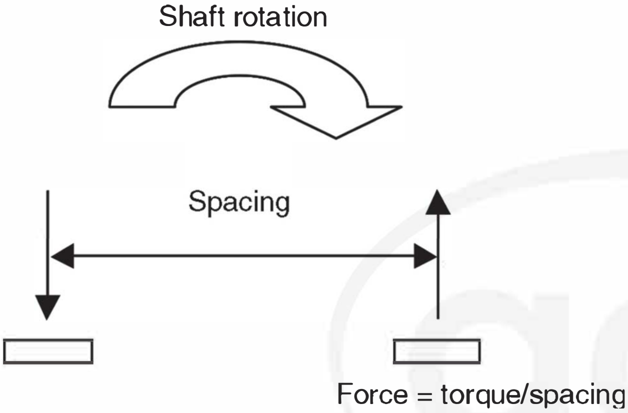


Fig. 4.2.3.1b—Equivalent forces for torque loads—torque resisted by longitudinal equipment soleplates.

The torque load is generally resolved into a vertical force couple by dividing it by the center-to-center distance between longitudinal soleplates or anchor points (Fig. 4.2.3.1b). When the machine is supported by transverse soleplates only, the torque is applied along the width of the soleplate assuming a straight line variation of force (Fig. 4.2.3.1c).

The torque on a generator stator is applied in the same direction as the rotation of the rotor and can be high due to startup or an electrical short circuit.

4.2.3.2 *Condenser vacuum loads*—For steam turbine-generator machines, usually a condenser is connected to the low-pressure (LP) turbine and may share one common foundation with the steam turbine-generator. The condenser generates vacuum loads on the steam turbine (LP turbine) when an expansion joint is provided between the condenser duct and the LP turbine exhaust neck. Condenser vacuum loads are normally based on zero absolute pressure. The vacuum loads can act vertically on the tabletop of the foundation if the condenser is located directly below the LP turbine, and it can also act laterally to the foundation if the condenser is not located directly below the LP turbine. The magnitude of the vacuum load can be estimated as the standard atmospheric pressure (14.7 psi [101.4 kPa]) times the

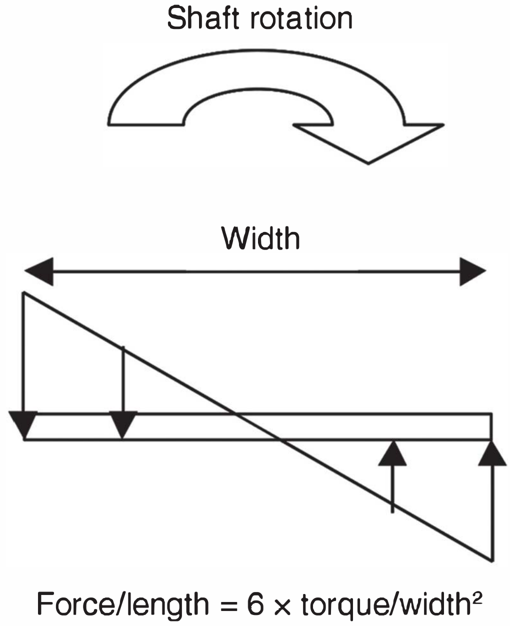


Fig. 4.2.3.1c—Equivalent forces for torque loads—torque resisted by transverse equipment soleplates.

neck duct opening area of the LP turbine. The vacuum loads are normally supplied by the machine manufacturer.

4.2.3.3 *Axial thrust loads*—Axial loads result from gas-flow momentum changes through gas turbines, or from the pressure differences between the gas inlet and exhaust. These loads generally are treated as a static load whose magnitude varies depending on the normal operation level exhaust of back pressure or shut-down level of exhaust back pressure. The axial thrust loads are normally supplied by the machine manufacturer.

4.2.3.4 *Piping loads*—Piping forces can be calculated based on the pipe stress analysis, taking into account the pipe weight (including insulation weight), fluid weight, fluid dynamics, thermal expansion/contractions, valve operations, and environmental load conditions (seismic, wind, and ice). These forces are transferred to the foundation by pipe supports, attachments, or anchors attached to the foundation along the piping system.

The piping load results from steam pipes and circulating water pipes attached to the turbine sections. The resultant piping loads acting on various points of support and guidance for the turbine casing reach the foundation as forces. During a valve trip, the turbine foundation can be subjected to significant piping valve trip loads. The piping loads usually are determined by the mechanical pipe engineers.

4.2.3.5 *Thermal loads*—Changing temperatures of machines and their foundations cause expansions, contractions, and distortions, causing the various parts to try to slide on the support surfaces. Thermal loads or self-straining loads may be divided into one of three loads types:

a) Thermal friction loads

b) Thermal expansion/contraction loads

c) Temperature change and temperature gradient change on the foundation member cross section

Thermal friction load occurs at the machine supports (at sole plates) from static friction resistance to the thermal movement of the machine components. The magnitude of the resulting frictional forces depends on the magnitude of the temperature change, the location of the supports, and on the condition of the support surfaces. The thermal forces do not impose a net force on the foundation to be resisted by supporting medium (soil or piles) because the forces on any surface are balanced by equal and opposite forces on other support surfaces. Thermal forces, however, may govern the design of the grout system, pedestals, and hold downs and may cause thermal stresses in the supporting concrete structures, including girders, slabs, and columns/pedestals.

It is not possible to exactly calculate the exact thermal loading because it depends on numerous factors, including distance between anchor points, magnitude of temperature change, the material properties, the condition of the sliding surface, and the magnitude of the vertical load on each soleplate; it is, therefore, approximated. The static friction resistance has to be overcome before thermal movements occurs and many engineers use this value for the design loads. The magnitude of the frictional load may be calculated as follows

$$\text{friction force} = (\text{friction coefficient}) \times (\text{normal load acting through soleplate}) \quad (4.2.3.5)$$

The friction coefficient generally varies from 0.2 to 0.5. Loads acting through the soleplate include machine dead load, normal torque load, anchor bolt load, and piping loads.

Normally, the expected thermal deflection at various bearings is estimated by the manufacturer based on past field measurements on existing units. The machine erector then compensates for the thermal deflection during installation.

Mandke and Smalley (1989, 1992) and Smalley (1985) illustrate the effects of thermal loads and deflections in the concrete foundation of a large reciprocating compressor and their influence on the machine.

Heat transfer to the foundation can be by convection across an air gap and by conduction through points of physical contact. The resultant temperature gradients induce deformations, strains, and stresses.

When evaluating thermal stress, the calculations are strongly influenced by the stiffness and restraint against deformation for the structural member in question. Therefore, it is important to consider the self-relieving nature of thermal stress due to deformation and concrete cracking to prevent being overly conservative in the analysis. As the thermal forces are applied to the foundation member by the machine, the foundation member changes length and thereby provides reduced resistance to the machine forces. This phenomenon can have the effect of reducing the thermal forces from the machine.

Accurate determinations of concrete surface temperatures and thermal gradients are also important. Under steady-state normal operating conditions, temperature distributions across structural sections are usually linear. The air gap between the machine casing and foundation provides a significant means for dissipating heat, and its effect should be included when establishing surface temperatures.

4.2.4 *Special loads for foundations*—The following special static load conditions are recommended in some proprietary standards for large equipment on foundations to ensure adequate strength and deflection control, especially if the appropriate information is not provided by the equipment manufacturer:

a) Vertical force equal to 50 percent of the total weight of each machine

b) Horizontal force (in the transverse direction) equal to 25 percent of the total weight of each machine

c) Horizontal force (in the longitudinal direction) equal to 10 percent of the total weight of each machine

These forces are additive to normal gravity loads and are considered to act at the centerline of the machine shaft. Loads a, b, and c are not considered to act concurrently with one another.

4.2.5 *Construction and maintenance loads*—Construction (erection and installation), hydro test, and maintenance loads are temporary loads required for installing or dismantling machine components during erection or maintenance. Erection loads are usually furnished in the manufacturer's foundation load drawing and should be used in conjunction with other specified dead, live, and environmental loads. Maintenance loads occur any time the equipment is being drained, cleaned, repaired, and realigned or when the components are being removed or replaced. Loads may result from maintenance equipment, davits, and hoists. Guidance on construction loads is provided in ASCE/SEI 37. Environmental loads can be reduced per ASCE/SEI 37 when combined with maintenance or construction loads.

A hydro-test load is the weight of water or other fluid that is placed temporarily into a vessel to confirm the integrity of the equipment.

4.2.6 *Machine catastrophic loads*—Machine catastrophic loads are caused by machine malfunction such as generator short circuit, turbine loss-of-blade/loss-of-bucket or a bowed rotor in a steam turbine. Catastrophic loads are generally applied at the bearings of the machine shaft vertically or horizontally and are not considered to act simultaneously with seismic loads, as there is a very low probability of these loads occurring at the same time. Often in the design for an electrical-short-circuit-induced torque load, the emergency drive torque is treated as an equivalent static load and the magnitude is determined by applying a magnification factor to the normal torque. Although these loads are time-dependent, they are normally given as equivalent static loads to simplify the foundation design efforts. These loads are normally provided by the machine manufacturer. Consultation with the generator manufacturer is necessary to establish the appropriate magnification factor.

4.3—Dynamic machine loads

4.3.1 *Rotary machine loads due to unbalanced masses*—These rotating loads are generated by the rotating motion of one or more impellers or rotors. Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the axis of rotation. In theory, it is possible to precisely balance the rotating elements of rotating machinery. In practice, this is never achieved; slight mass eccentricities always remain. During operation, the eccentric rotating mass produces centrifugal forces that are proportional to the square of machine speed. Centrifugal forces generally increase during the design service life of the machine due to conditions such as machine wear, rotor play, and dirt accumulation.

A rotating machine transmits dynamic force to the foundation predominantly through its bearings (with small, generally unimportant exceptions such as seals and the air gap in a motor). The forces acting at the bearings are a function of the level and axial distribution of the unbalance, the geometry of the rotor and its bearings, the speed of rotation, and the dynamic characteristics of the rotor-bearing system. At or near a critical speed, the force from rotating unbalance can be substantially amplified, sometimes by a factor of five or more due to resonance.

The engineer should request the machine manufacturer to provide the following information:

a) Design levels of unbalance and basis—This information defines the unbalance level that is the basis for calculating the subsequent transmitted forces.

b) Dynamic forces transmitted to the bearing pedestals under the following conditions:

1. Unbalance levels over operating speed range
2. At highest vibration when running at critical speeds
3. At a vibration level where the machine is just short of tripping due to high vibration
4. Maximum level of upset condition the machine is designed to survive (for example, loss of one or more blades)

Items 1 and 2 document the predicted dynamic forces resulting from levels of unbalance assumed in design for normal operation. Using these forces, it is possible to predict the normal dynamic vibration of the machine and its foundation.

Item 3 identifies a maximum level of transmitted force under which the machine could operate continuously without tripping; the foundation should have the strength to tolerate such a dynamic force on a continuous basis.

Item 4 identifies the higher level of dynamic forces that could occur under occasional upset conditions over a short period of time. If the machine is designed to tolerate this level of dynamic force for a short period of time, then the foundation should also be designed to tolerate it for a similar period.

If an independent dynamic analysis of the rotor-bearing system is performed by the manufacturer, the end user, or a third party, the results likely provide some or all the above dynamic forces transmitted to the foundation.

By assuming that the dynamic force transmitted to the bearings equals the rotating unbalanced force generated by the rotor, information on unbalance can be used to estimate or confirm the transmitted force.

4.3.1.1 *Machine unbalance load provided by manufacturer*—When the mass unbalance (eccentricity) is known or stated by the manufacturer, the resulting dynamic force amplitude is

$$F_o = m_r e_m \omega_o^2 S_f \quad \text{(lbf)}$$
$$F_o = m_r e_m \omega_o^2 S_f / 1000 \quad \text{(N)} \qquad (4.3.1.1)$$

where F_o should be distributed to the bearings considering the rotor/shaft span between bearings as a simple beam and the center of mass of the rotor location within the span.

4.3.1.2 *Machine unbalance load (meeting industry criteria)*—Many rotating machines are balanced to an initial balance quality either in accordance with the manufacturer's procedures or as specified by the purchaser. ISO 1940-1 and ASA/ANSI S2.19 define balance quality in terms of a constant $e_m\omega_o$. For example, the normal balance quality Q for parts of process-plant machinery is 0.25 in./s (6.3 mm/s). Typical balance quality grade examples are shown in Table 4.3.1.2. To meet these criteria, a rotor intended for faster speeds should be better balanced than one operating at a slower speed. Using this approach, Eq. (4.3.1.1) can be rewritten as

$$F_o = m_r Q \omega_o S_f \quad \text{(lbf)}$$
$$F_o = m_r Q \omega_o S_f / 1000 \quad \text{(N)} \qquad (4.3.1.2a)$$

where Q is in inch-pound and SI systems is ft/s and mm/s, respectively.

API 617 and API 684 work with maximum residual unbalance U_{max} criteria for petroleum processing applications. The mass eccentricity is determined by dividing U_{max} by the rotor weight. For axial and centrifugal compressors with maximum continuous operating speeds greater than 25,000 rpm, API 617 establishes a maximum allowable mass eccentricity of 10×10^{-6} in. (254 nm). For compressors operating at slower speeds, the maximum allowable mass eccentricity is

$$e_m = 0.25/f_o \quad \text{(in.)}$$
$$e_m = 6.3/f_o \quad \text{(mm)} \qquad (4.3.1.2b)$$

where f_o is the less than or equal to 25,000 rpm.

This permitted initial mass eccentricity equates to ISO 1940-1 balance quality Grade G0.7, which is much smaller than the G2.5 that would be applied to this type of equipment (Table 4.3.1.2, turbo compressors). As such, the dynamic force calculated from this API 617 consideration will be quite small and a larger service factor might be used to calculate a realistic design force.

API 617 also identifies a limitation on the peak-to-peak vibration amplitude $((12{,}000/f_o)^{0.5}$ mil $[25.4(12{,}000/f_o)^{0.5}$ μm]) during mechanical testing of the compressor with the equipment operating at its maximum continuous speed. Some design firms use this criterion as a mechanical test vibration limit for the mass eccentricity e_m, using Eq. (4.3.1.2a), and a service factor S_f of 2.0 to calculate the zero to peak dynamic force amplitude as

Table 4.3.1.2—Balance quality grades for selected groups of representative rigid rotors*

Balance quality guide	Product of $e\omega$, in./s (mm/s)	Rotor types—general examples
G1600	63 (1600)	Crankshaft/drives of rigidly mounted, large, two-cycle engines
G630	25 (630)	Crankshaft/drives of rigidly mounted, large, four-cycle engines
G250	10 (250)	Crankshaft/drives of rigidly mounted, fast, four-cylinder diesel engines
G100	4 (100)	Crankshaft/drives of fast diesel engines with six or more cylinders
G40	1.6 (40)	Crankshaft/drives of elastically mounted, fast four-cycle engines (gasoline or diesel) with six or more cylinders
G16	0.6 (16)	Parts of crushing machines; drive shafts (propeller shafts, cardan shafts) with special requirements; crankshaft/drives of engines with six or more cylinders under special requirements
G6.3	0.25 (6.3)	Parts of process plant machines; centrifuge drums, paper machinery rolls, print rolls; fans; flywheels; pump impellers; machine tool and general machinery parts; medium and large electric armatures (of electric motors having at least 3-1/4 in. [80 mm] shaft height) without special requirement
G2.5	0.1 (2.5)	Gas and steam turbines, including marine main turbines; rigid turbo-generator rotors; turbo-compressors; machine tool drives; medium and large electric armatures with special requirements; turbine driven pumps
G1	0.04 (1)	Grinding machine drives
G0.4	0.015 (0.4)	Spindles, discs, and armatures of precision grinders

*Excerpted from ASA/ANSI S2.19.

$$F_o = \frac{W_r f_o^{1.5}}{322{,}000} \quad (4.3.1.2c)$$

Units of F_o and W_r are lbf and N in inch-pound and SI systems, respectively.

4.3.1.3 *Machine unbalance load (determined from a formula)*—Rotating machine manufacturers often do not report the unbalance that remains after balancing. Consequently, formulas are frequently used to ensure that foundations are designed for some minimum unbalance, which generally includes some allowance for unbalance increases over time. One general-purpose method assumes that balancing improves with machine speed and that there is a linear relationship between the unbalanced forces and the machine speed. The zero-to-peak centrifugal force amplitude from Eq. (4.3.1.2a) and (4.3.1.2b) and a service factor of S_f of 2.5 from one such commonly used expression is

$$F_o = \frac{W_r f_o}{6000} \quad (4.3.1.3)$$

where units of F_o and W_r are lbf and N in inch-pound and SI systems, respectively.

Equations (4.3.1.1), (4.3.1.2a), (4.3.1.2c), and (4.3.1.3) appear to be very different: the exponents on the speed of rotation vary from 1 to 1.5 to 2, constants vary widely, and different variables appear. Some equations use mass, others use weight. In reality, the equations are more similar than they appear. Given the right understanding of Q as a replacement for $e_m\omega_o$, Eq. (4.3.1.1), (4.3.1.2a), and (4.3.1.3) take on the same character. These equations then indicate that the design force at operating speed varies linearly with both the mass of the rotating body and the operating rotational speed. Once that state is identified, Eq. (4.3.1.1) can be adjusted to reflect the actual speed of rotation, and the dynamic centrifugal force is seen to vary with the square of the speed. Restating Eq. (4.3.1.2c) and (4.3.1.3) in the form of Eq. (4.3.1.1) allows for the development of an effective eccentricity implied within these equations with the comparison shown in Fig. 4.3.1.3. Equation (4.3.1.3) produces the same result as Eq. (4.3.1.2a) using Q = 0.25 in./s (6.3 mm/s) and S_f = 2.5.

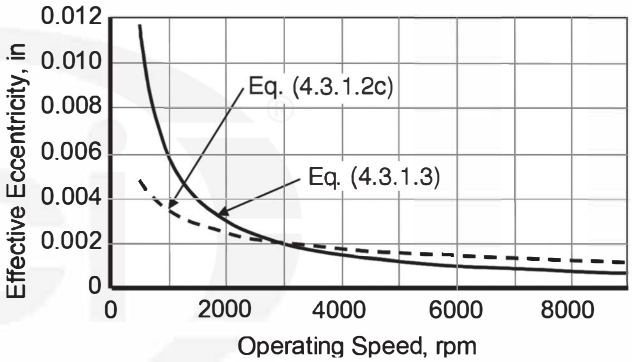


Fig. 4.3.1.3—Machine unbalanced loads—comparison of effective eccentricity.

The centrifugal forces due to mass unbalance are considered to act at the center of gravity of the rotating part and vary harmonically at the speed of the machine in the two orthogonal directions perpendicular to the shaft axis. The forces in the two orthogonal directions are equal in magnitude and 90 degrees out of phase, and are transmitted to the foundation through the bearings. Schenck (1990) provides useful information about balance quality for various classes of machinery.

4.3.1.4 *Machine unbalance load (determined from trip vibration level and effective bearing stiffness)*—Because a rotor is often set to trip off at high vibration, it can be expected to operate continuously at any vibration level up to the trip limit. Given the effective bearing stiffness, it is possible to calculate the maximum dynamic force amplitude as

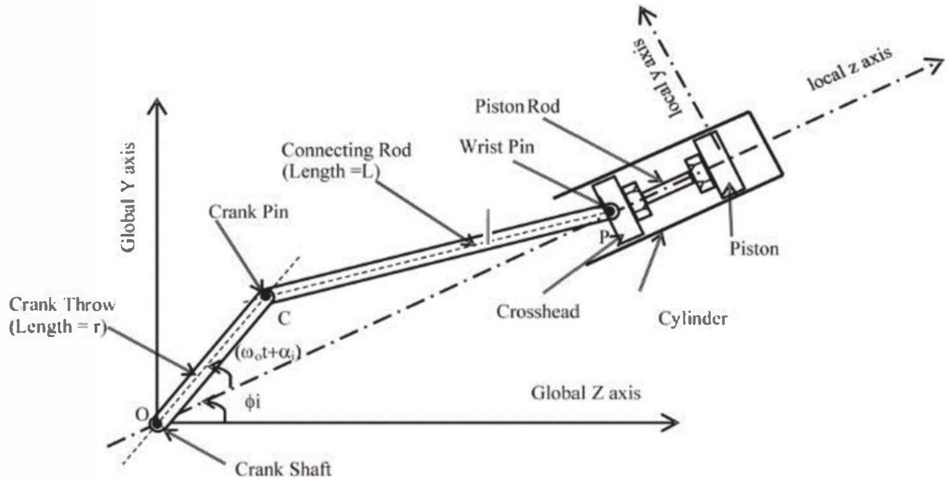


Fig. 4.3.2.1—Single crank mechanism.

$$F_o = V_{max}K_{eff} \quad (4.3.1.4)$$

To use this approach, the manufacturer should provide effective bearing stiffness from the bearing geometry and operating conditions (such as viscosity and speed) or provide the corresponding forces for the foundation design.

4.3.1.5 *Steam or gas turbine generator units*—The determination of the transmitted dynamic rotating unbalanced forces from the bearings to the foundation, for large rotating machines such as steam turbine generator units and combustion turbine generator units with complicated rotor configuration, shaft configurations, or both, can be completed by a dynamic analysis of the rotor-bearing system (the rotor/shaft power train dynamic analysis). This dynamic analysis should be completed under varying conditions of unbalance and at varying speeds. This dynamic analysis is based on the expected level of rotor/shaft unbalance that corresponds to a turbine generator rotor in operable condition but in need of balancing. The analysis can be completed by using an appropriate combination of computer programs for calculating bearing dynamic characteristics and the response of the rotor in the bearings due to the unbalance and the operating speeds. Such an analysis would usually be performed by the machine manufacturer. Results of such analyses, especially values for transmitted bearing forces, represent the best source of information for use by the engineer responsible for foundation design.

4.3.1.6 *Loads from multiple rotating machines*—If a foundation supports multiple rotating machines, the engineer should calculate the unbalanced dynamic force based on the mass, unbalance, and operating speed of each rotating component. The response to each rotating mass is then combined to determine the total response. Some practitioners, depending on the specific situation of machine size and criticality, find it advantageous to combine the unbalanced forces from each rotating component into a single resultant unbalanced force. The method of combining two dynamic forces is up to individual judgment and often involves some approximations. In some cases, loads or responses can be added absolutely. In other cases, the loads are treated as out-of-phase so that twisting effects are increased. Often, the operating speed of the equipment should be considered. Even if operating speeds are nominally the same, the engineer should recognize that during normal operation, the speed of the machines will vary and beating effects can develop. Beating effects develop as two machines operate at close to the same speed. At one point in time, responses to the two machines are additive and motions are maximized. A short time later, the responses cancel each other and the motions are minimized. The net effect is a continual cyclic rising and falling of motion.

4.3.2 *Reciprocating machine loads*—Internal-combustion engines, piston-type compressors and pumps, some metal forming presses, steam engines, and other machinery are characterized by the rotating motion of a master crankshaft and the linear reciprocating motion of connected pistons or sliders. The motion of these components causes cyclically varying forces, often called reciprocating forces.

4.3.2.1 *Primary and secondary reciprocating loads*—The simplest type of reciprocating machine uses a single crank mechanism, as shown in Fig. 4.3.2.1. The idealization of this mechanism consists of a piston that moves within a guiding cylinder, a crank throw of length r that rotates about a crank shaft, and a connecting rod of length L. The connecting rod is attached to the piston at Point P and to the crank at Point C. The Wrist Pin P oscillates while the Crank Pin C follows a circular path. This idealized single cylinder illustrates the concept of a machine producing both primary and secondary reciprocating forces.

If the crank is assumed to rotate at a constant angular velocity ω_o, the translational acceleration of the piston along its axis may be evaluated. If z_p is defined as the piston displacement toward the crankshaft (local z-axis), an expression can be written for z_p at any time t. Further, the velocity and acceleration can also be obtained by taking the first and second derivatives of the displacement expression with respect to time. The displacement, velocity, and acceleration expressions for the motion of the piston are as follows.

$$z_p = \left(r + \frac{r^2}{4L}\right) - r\left(\cos\omega_o t + \frac{r}{4L}\cos 2\omega_o t\right) \quad (4.3.2.1a)$$

$$\dot{z}_p = r\omega_m\left(\sin\omega_o t + \frac{r}{2L}\sin 2\omega_o t\right) \quad (4.3.2.1b)$$

$$\ddot{z}_p = r\omega_m^2\left(\cos\omega_o t + \frac{r}{L}\cos 2\omega_o t\right) \quad (4.3.2.1c)$$

Note that the expressions contain two terms, each with a sine or cosine; the term that varies with the frequency of the rotation, ω_o, is referred to as the primary term whereas the term that varies at twice the frequency of rotation, $2\omega_o$, is called the secondary term.

Similar expressions can be developed for the motion of the rotating parts of the crank along the local z-axis (parallel to piston movement) and local y-axis (perpendicular to piston movement). If any unbalance in the crankshaft is replaced by a mass concentrated at the Crank Pin C, such that the inertia forces are the same as in the original system, the following terms for motion at Point C can be written.

$$y_c = r\sin\omega_o t \quad (4.3.2.1d)$$

$$\dot{y}_c = r\omega_m\cos\omega_o t \quad (4.3.2.1e)$$

$$\ddot{y}_c = -r\omega_o^2\sin\omega_o t \quad (4.3.2.1f)$$

$$z_c = r(1 - \cos\omega_o t) \quad (4.3.2.1g)$$

$$\dot{z}_c = r\omega_o\sin\omega_o t \quad (4.3.2.1h)$$

$$\ddot{z}_c = r\omega_o^2\cos\omega_o t \quad (4.3.2.1i)$$

Identifying a part of the connecting rod (usually one-third of its mass) plus the piston as the reciprocating mass m_{rec} concentrated at Point P and designating the remainder of the connecting rod plus the crank as the rotating mass m_{rot} concentrated at Point C, expressions for the unbalanced forces are as follows.

Force parallel to piston movement:

$$F_z = (m_{rec} + m_{rot})r\omega_o^2\cos\omega_o t + m_{rec}\frac{r^2\omega_o^2}{L}\cos 2w_o t \quad (4.3.2.1j)$$

Force perpendicular to piston movement:

$$F_y = m_{rot}r\omega_o^2\sin\omega_o t \quad (4.3.2.1k)$$

Note that Eq. (4.3.2.1j) consists of two terms, a primary force:

$$(m_{rec} + m_{rot})r\omega_o^2\cos\omega_o t \quad (4.3.2.1l)$$

and a secondary force:

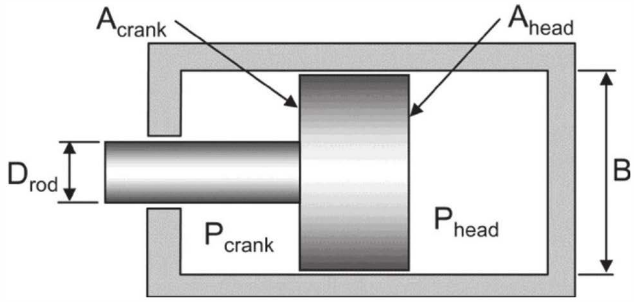


Fig. 4.3.2.2—Schematic diagram of double-acting compressor cylinder and piston.

$$m_{rec}\frac{r^2\omega_o^2}{L}\cos 2\omega_o t \quad (4.3.2.1m)$$

whereas Eq. (4.3.2.1k) has only a primary component.

4.3.2.2 *Compressor gas loads*—A reciprocating compressor raises the pressure of a gas flow by imparting reciprocating motion on a piston within a cylinder. The piston normally compresses gas during both directions of reciprocating motion. As gas flows to and from each end, the pressure of the gas increases as it is compressed by each stroke of the piston. The increase in pressure within the cylinder creates reaction forces on the head and crank ends of the piston that alternate as gas flows to and from each end of the cylinder.

The gas force applied to the piston rod equals the instantaneous difference between the pressure force acting on the head and crank end of the piston, as shown in Fig. 4.3.2.2 for typical single crank reciprocating machines.

The following formulation can be used to estimate the maximum force acting on the piston rod of an individual double-acting cylinder.

$$F_{rod} = [(P_{head})(A_{head}) - (P_{crank})(A_{crank})]F_1 \quad (4.3.2.2a)$$

$$A_{head} = (\pi/4)B_c^2 \quad (4.3.2.2b)$$

$$A_{crank} = (\pi/4)(B_c^2 - D_{rod}^2) \quad (4.3.2.2c)$$

The head and crank end pressures vary continuously and the differential force takes both positive and negative net values during each cycle of piston motion. The normal approach is to establish the head and crank pressures using the maximum and minimum suction and discharge pressures. For design purposes, it is common to multiply Eq. (4.3.2.2a) by a factor F_1 to help account for the natural tendency of gas forces to exceed the values based directly on suction and discharge pressures due to flow resistances and pulsations. Machines with good pulsation control and low external flow resistance may achieve F_1 as small as 1.1. For machines with low compression ratio, high pulsations, or highly resistive flow through piping and nozzles, F_1 can approach 1.5 or even higher. A reasonable working value for F_1 is 1.15 to 1.2.

Preferably, the maximum rod force resulting from gas pressures is based on knowledge of the continuous variation of pressure in the cylinder (measured or predicted). In a repair situation, measured cylinder pressure variation using a cylinder analyzer provides the most accurate value of gas forces. Even without cylinder pressure analysis, extreme operating values of suction and discharge pressure for each stage should be recorded before the repair and used in Eq. (4.3.2.2a).

On new compressors, the engineer should ask the machine manufacturer to provide values for maximum compressive and tensile gas loads on each cylinder rod and to recommend a value of F_1 if these forces are based on suction and discharge pressures.

Gas forces act on the crankshaft with an equal and opposite reaction to the force on the cylinder. Thus, crankshaft and cylinder forces globally balance each other. Between the crankshaft and the cylinder, however, the metal compressor frame stretches or contracts in tension or compression under the action of the gas forces. The forces due to frame deflections are transmitted to the foundation through connections with the compressor frame. When acting without slippage, the frame and foundation become an integral structure and together stretch or contract under the gas loads.

The magnitude of gas force transferred into the foundation depends on the relative flexibility of the compressor frame. A very stiff frame transmits only a small fraction of the gas force while a very flexible frame transmits most or all the force. Similar comments apply to the transfer of individual cylinder inertia forces.

Based on limited comparisons using finite element analysis (Smalley 1988), the following guidelines are suggested for gas and inertia force loads transmitted to the foundation by a typical compressor

$$F_{block} = F_{rod}/F_{red} \quad (4.3.2.2d)$$

$$(F_{bolt})_{CHG} = [(F_{rod})/(N_{bolt})_{CHG}]/F_{red} \quad (4.3.2.2e)$$

$$(F_{bolt})_{frame} = [(F_{unbalance}/(N_{bolt})]/F_{red} \quad (4.3.2.2f)$$

The factor F_{red} is used to simplify a complex problem, thus avoiding the application of unrealistically high loads on the anchor bolts and the foundation block. The mechanics involved in transmitting loads are complex and cannot easily be reduced to a simple relationship between a few parameters beyond the given load equations. A detailed finite-element analysis of metal compressor frame, chock mounts, concrete block, and grout will account for the relative flexibility of the frame and its foundation in determining individual anchor bolt loads and implicitly provide a value for F_{red}. If the frame is very stiff relative to the foundation, the value for F_{red} will be higher, implying more of the transmitted loads are carried by the frame and less by the anchor bolts and foundation block. Based on experience, a value of 2 for this factor is conservatively low; however, higher values have been seen with frames designed to be especially stiff.

Simplifying this approach, Smalley and Harrell (1997) suggest using a finite element analysis to calculate forces transmitted to the anchor bolts. If a finite element analysis is not possible, the engineer should obtain from the machine manufacturer or calculate the maximum horizontal gas force and maximum horizontal inertia force for any throw or cylinder. The mounts, anchor bolts, and blocks are then designed for

$$F_{THROW} = (\text{greater of } F_{GMAX} \text{ or } F_{IMAX})/2 \quad (4.3.2.2g)$$

4.3.2.3 *Reciprocating inertia loads for multi-cylinder machines*—As a practical matter, most reciprocating machines have more than one cylinder, and manufacturers arrange the machine components in a manner that minimizes the net unbalanced forces. For example, rotating parts such as the crankshaft can be balanced by adding or removing correcting weights. Translating parts such as pistons and those that exhibit both rotation and translation such as connecting rods can be arranged in such a way as to minimize the unbalanced forces and moments generated. Seldom, if ever, is it possible to perfectly balance reciprocating machines.

The forces generated by reciprocating mechanisms are functions of the mass, stroke, piston arrangement, connecting rod size, crank throw orientation (phase angle), and the mass and arrangement of counterweights on the crankshaft. For this reason, calculating the reciprocating forces for multi-cylinder machines can be quite complex and are therefore normally provided by the machine manufacturer. If the machine is an integral engine compressor, it can include, in one frame, cylinders oriented horizontally, vertically, or in between, all with reciprocating inertias.

Some machine manufacturers place displacement transducers and accelerometers on strategic points on the machinery. They can then measure displacements and accelerations at those points for several operational frequencies to determine the magnitude of the unbalanced forces and couples for multi-cylinder machines.

4.3.2.4 *Estimating reciprocating inertia forces from multi-cylinder machines*—In cases where the manufacturer's data are unavailable or components are being replaced, the engineer should use hand calculations to estimate the reciprocating forces from a multi-cylinder machine. One such procedure for a machine having n number of cylinders is discussed by Mandke and Troxler (1992).

4.3.3 *Impulsive machine loads*—The impulsive load generated by a forging hammer is caused by the impact of the hammer ram onto the hammer anvil. This impact process transfers the kinetic energy of the ram into kinetic energy of the entire hammer assembly. The post-impact velocity of the hammer is represented by

$$v_h = \frac{M_r}{M_h}(1+\alpha_h)v_r \quad (4.3.3a)$$

General experience indicates that α_h is approximately 60 percent for many forging hammer installations. From that point, the hammer foundation performance can be assessed as a rigid body oscillating as a single degree-of-freedom system with an initial velocity of v_h.

For metal-forming presses, the dynamic forces develop from two sources: the mechanical movement of the press components and material-forming process. Each of these forces is unique to the press design and application and needs to be evaluated with proper information from the press manufacturer and the owner.

The press mechanics often include rotating and reciprocating components. The dynamic forces from these individual pieces follow the rules established in earlier sections of this document for rotating and reciprocating components. Only the press manufacturer familiar with all the internal components can knowledgeably calculate the specific forces. Figure 4.3.1.3 presents a horizontal force time-history for a forging press. Similar presses can be expected to have similar characteristics; however, the specific values and timing data differ.

The press drive mechanisms include geared and direct-drive systems. Depending on the design, these drives may or may not be balanced. The press slide travels vertically through a set stroke of 1/2 in. (12 mm) to several inches (millimeters) at a given speed. Some small presses may have inclinable beds so that the slide is not moving vertically. It is often adequate to assume that the slide moves in a vertical path defined by a circularly rotating crankshaft; that is,

$$d_s(t) = \frac{S}{2}\sin(\omega_o t) \qquad (4.3.3b)$$

This leads to a dynamic inertia force from the slide of

$$F_s(t) = m_s\omega_o^2\frac{S}{2}\sin(\omega_o t) \text{ (lbf)}$$
$$F_s(t) = m_s\omega_o^2\frac{S}{2}\sin(\omega_o t)/1000 \text{ (N)} \qquad (4.3.3c)$$

This assumption is based on simple circular motions and simple linkages. Other systems may be in place to increase the press force and improve the timing. These other systems may increase the acceleration of the unbalanced weights and thus alter the magnitude and frequency components of the dynamic force transmitted to the foundation.

4.4—Environmental loads

4.4.1 *Wind loads*—Loads due to wind on the surface areas of the machine, auxiliary equipment, and the support foundation are based on the design wind speed for the particular site and are normally calculated in accordance with the governing local code or standard. Wind loads rarely govern the design of machine foundations except, perhaps, when the machine is located in an enclosure that is also supported by the foundation.

When designing machine foundations and support structures, most practitioners use the wind load provisions of ASCE/SEI 7. The analytical procedure of ASCE/SEI 7 provides wind pressures and forces for use in the design of the main wind-force-resisting systems and anchorage of machine components.

Most structural systems involving machines and machine foundations are relatively stiff (natural frequency in the lateral direction greater than 1 Hz). Consequently, the systems can be treated as rigid with respect to the wind gust effect factor, and simplified procedures can be used. If the machine is supported on flexible isolators and is exposed to the wind, the rigid assumption may not be reasonable and more elaborate treatment of the gust effects is necessary as described in ASCE/SEI 7 for flexible structural systems.

Appropriate consideration of the exposure conditions and importance factors is also recommended to be consistent with the facilities' requirements.

4.4.2 *Seismic loads*—Machinery foundations located in seismically active regions are analyzed for seismic loads. These loads should be determined as equivalent static loads or dynamic loads per applicable codes and standards.

Building codes and standards such as the International Building Code (International Code Council 2015), ASCE/SEI 7, FEMA P-750, and SEAOC Blue Book (2009) contain provisions for design of nonstructural components, including dynamic machinery. The seismic loads and design requirements for nonbuilding structures, including elevated tabletop-type foundation pedestals, are also contained in the provisions of these codes. For machinery supported above grade or on more flexible elevated pedestals, seismic amplification factors are also specified. For where isolators are used (such as spring/damper/absorber configurations) to support the machine, ASCE/SEI 7 provides criteria for seismic load calculation and design.

Depending on the weight of the machine with respect to the total weight of the machine-foundation system, the design seismic forces of the machine (nonbuilding structure) can be determined either by treating the machine weight as an integral part of the foundation weight (ASCE/SEI 7), or by treating the machine as a nonstructural component attached to the foundation (ASCE/SEI 7). For the seismic design of tabletop foundations, the foundation is usually treated as a supporting structure that should be designed in accordance with the requirements of ASCE/SEI 7.

For most heavy machine foundations, it is common to treat the machine weight as an integral part of the foundation for determining machine seismic load effects. In the seismic weight calculation, the full or percentage of the live load should be used per ASCE/SEI 7.

Horizontal and vertical components of seismic loads should be investigated in the load conditions and combinations in the following sections per ASCE/SEI 7.

4.4.3 *All other loads*—Environmental loads such as earth, water, rain, snow, and ice should be included in the design per applicable building codes.

4.5—Load conditions

During their lives, machinery equipment support structures and foundations undergo different load conditions, including construction, testing, shutdown, maintenance, and normal and abnormal operation. For each loading condition, there can be one or more combinations of loads that apply

Table 4.6—Load classifications for ultimate strength design

Design loads	Load classification
Weight of structure, equipment, internals, insulation, and platforms Fluid loads during testing and operation Anchor and guide loads	Dead
Platform and walkway loads Materials to be temporarily stored during maintenance Materials normally stored during operation such as tools and maintenance equipment Vibrating equipment forces Impact loads for hoist and equipment handling utilities	Live
Seismic loads Snow, ice, or rain loads Wind loads	Environmental

Notes: Thermal or self-straining loads classifications should be evaluated per requirements of ASCE/SEI 7.

Load due to lateral earth pressure, ground water pressure, or pressure of the bulk materials classifications should be evaluated per guidelines of ASCE/SEI 7.

to the structure or foundation. The following load conditions are generally considered in design:

a) Construction condition represents the design loads that act on the structure/foundation during its construction.

b) Testing condition represents the design loads that act on the structure/foundation while the equipment being supported is undergoing testing, such as hydro test.

c) Normal operating condition represents the design loading during periods of normal equipment operation.

d) Abnormal operating condition represents the design loading during periods when unusual or extreme operating loads act on the structure/foundation.

e) Maintenance condition (empty or shutdown) represents the design loads that act on the structure when the supported equipment is at its least weight due to removal of process fluids, applicable internals, or both, as a result of maintenance or other out-of-service disruption.

4.6—Load combinations

Table 4.6 shows the general classification of loads for use in determining the applicable load factors in strength design (ACI 318; ASCE/SEI 7; ASCE/SEI 37). In considering soil stresses or loading on piles, the normal approach is working stress design without load factors and with overall factors of safety identified as appropriate by geotechnical engineers. The load combinations frequently used for the various load conditions are as follows:

a. Construction
   i. Dead loads + construction loads
   ii. Dead loads + construction loads + reduced wind loads + snow, ice, or rain loads
   iii. Dead loads + construction loads + seismic loads + snow, ice, or rain loads
b. Hydro testing
   i. Dead loads + test loads
   ii. Dead loads + test loads + live loads + snow, ice, or rain loads
   iii. Dead loads + test loads + reduced wind loads + snow, ice, or rain loads
c. Normal operation
   i. Dead loads + machine operation loads + live loads (for multi bearings machine post-alignment deflection check)
   ii. Dead loads + machine operation loads + live loads + wind loads + snow, ice, or rain loads
   iii. Dead loads + machine operation loads + seismic loads + snow, ice, or rain
d. Abnormal operation
   i. Dead loads + upset (abnormal, emergency, catastrophic) machine loads + live loads + reduced wind loads
e. Maintenance loads
   i. Dead loads + maintenance loads + live loads + snow, ice, wind or rain loads

Environmental loads can be reduced per ASCE/SEI 37 when combined with maintenance or construction loads due to the low probability of the loads occurring at the same time.

CHAPTER 5—IMPEDANCE OF THE SUPPORTING MEDIUM

5.1—Overview and use of soil impedance

The foundation dynamic response depends on the stiffness and damping characteristics of the machine-foundation-soil system. This section presents a general introduction to this subject and a summary of approaches and formulas often used to evaluate the stiffness and damping of both soil-supported and pile foundations. These stiffness and damping relationships, collectively known as impedance, are used for determining both free-vibration performance and motions of the foundation system due to the dynamic loading associated with the machine operation.

The simplest mathematical model used in dynamic analysis of machine-foundation systems is a single-degree-of-freedom representation of a rigid mass vertically supported on a single spring and damper combination (Fig. 5.1(a)). This model is applicable if the center of gravity of the machine-foundation system is directly over the center of soil resistance and the resultant of the dynamic forces (acting through a center of force [CF]) is a vertical force passing through the center of gravity. The vertical impedance (k_v^*) of the supporting medium is necessary for this model.

The next level of complexity is a two-degree-of-freedom model commonly used when lateral dynamic forces act on the system (Fig. 5.1(b)). Because the lines of action of the applied forces and the soil resistance do not coincide, the rocking and translational motions of the system are coupled. For this model, the engineer needs to calculate the horizontal translational impedance (k_u^*) and the rocking impedance ($k_ψ^*$) of the supporting medium. These impedance values, especially the rocking terms, are usually different about different horizontal directions.

Application of the lateral dynamic forces can cause a machine-foundation system to twist about a vertical axis. To model this behavior, the engineer needs to determine the torsional impedance (k_η^*) of the supporting medium. As in the vertical model, if the in-plan eccentricities between the center of gravity and center of soil resistance are small, this analysis is addressed with a single-degree-of-freedom representation.

When the machine-foundation system is infinitely rigid and on the surface, the impedance (k_{u1}^*, k_{u2}^*, k_v^*, $k_{\psi 1}^*$, $k_{\psi 2}^*$, k_η^*) can be represented as six frequency-dependent spring-mass systems (three translational and three angular) and six frequency-dependent viscous or hysteretic dampers (three translational and three angular) that act at the centroid of the contact area between the foundation and the supporting medium (Fig. 5.1(c)). If the support is provided by piles or isolators, the equivalent springs and dampers act at the center of stiffness of those elements. When the foundation is embedded, additional terms can be introduced as spring and damper elements acting on the sides of the foundation. The effective location of the embedment impedance is determined rationally based on the character of the embedment (Beredugo and Novak 1972; Novak and Beredugo 1972; Novak and Sachs 1973; Novak and Sheta 1980; Lakshmanan and Minai 1981; Gazetas 1991).

The next level of complexity addresses flexible foundations. The structural model of these foundations is typically created using finite element techniques. The flexibility of the structure might be modeled with beam-type elements, two-dimensional plane stress elements, plate bending elements, three-dimensional solid elements, or some combination of these.

The supporting medium for these flexible foundations is addressed in one of three basic manners. The simplest approach uses a series of translational springs at each contact point between the structure and the soil. These spring parameters are calculated from the spring constants determined considering portions of the mat foundation small enough to be assumed rigid (Arya et al. 1979) or by some similar approximated procedure. The other two approaches use the finite element method, boundary element method, or both, for calculating the dynamic impedance matrix of the supporting medium. Such approaches are typically used for major, complex situations where a high degree of refinement is justified (for example, complex soil stratigraphy, irregular foundation geometry, or both). A detailed discussion of these techniques is outside the scope of this report.

Nevertheless, the fundamental principles and concepts discussed herein for infinitely rigid foundation also apply to flexible foundations. In summary, the main difference between flexible and rigid foundations is that flexible foundations are modeled using finite or boundary element methods, which leads to a dynamic impedance matrix rather than six dynamic impedances. This matrix will contain all degrees of freedom associated with nodes located at the boundary between the foundation and its support.

5.2—Basic dynamic concepts

The basic mathematical model used in the dynamic analysis of various infinitely rigid machine-foundation systems is a lumped mass with a spring and dashpot (Fig. 5.2a). This lumped mass includes the machine mass, foundation mass,

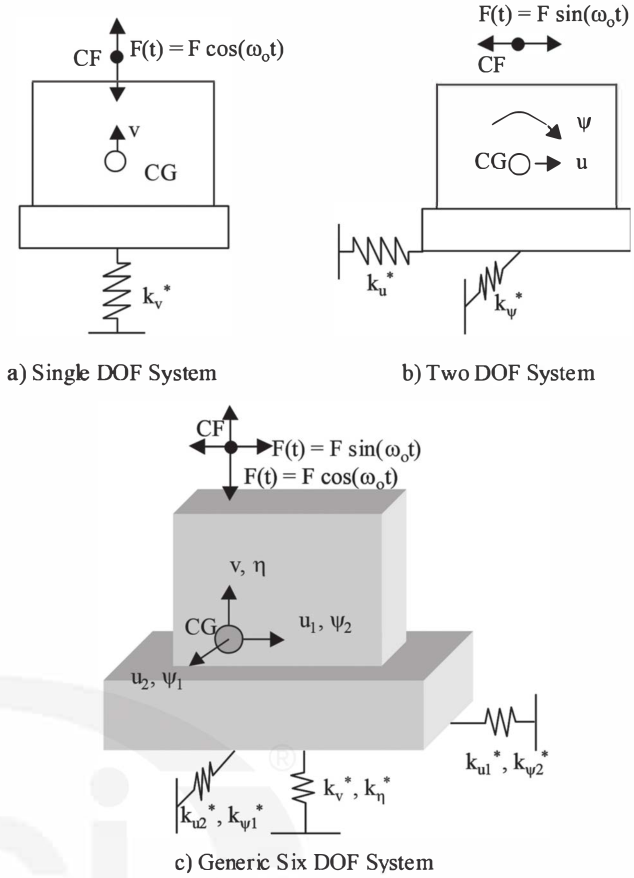


Fig. 5.1—Common dynamic models.

and often an added soil mass. A mass m, free to move only in one direction—for example, vertical—is said to have only a single degree of freedom (SDOF) (Fig. 5.1(a)). The behavior of the mass depends on the nature of both the spring and the dashpot.

The spring, presumed to be massless, represents the elasticity of the system and is characterized by the stiffness constant k. The stiffness constant is defined as the force that produces a unit displacement of the mass. For a general displacement u of the mass, the force in the spring (the restoring force) is ku.

In dynamics, the displacement varies with time t, thus, $u = u(t)$. Because the spring is massless, the dynamic stiffness constant k is equal to the static constant k_{st}, and, thus, the restoring force ku and k are independent of the rate or frequency at which the displacement varies.

The same concept of stiffness can be applied to a harmonically vibrating column with mass uniformly distributed along its length (Fig. 5.2b(a)). For an approximate analysis at low frequency, the distributed column mass can be replaced by a concentrated (lumped) mass m_s, attached to the top of the column, and the column itself can be considered massless (Fig. 5.2b(b)). Consequently, a static stiffness constant k_{st}, independent of frequency, can be used to describe the stiffness of this massless column. The elastic force in the column just below the mass is $k_{st}u$ for any displacement u. Nevertheless, the total restoring force generated by the column at the

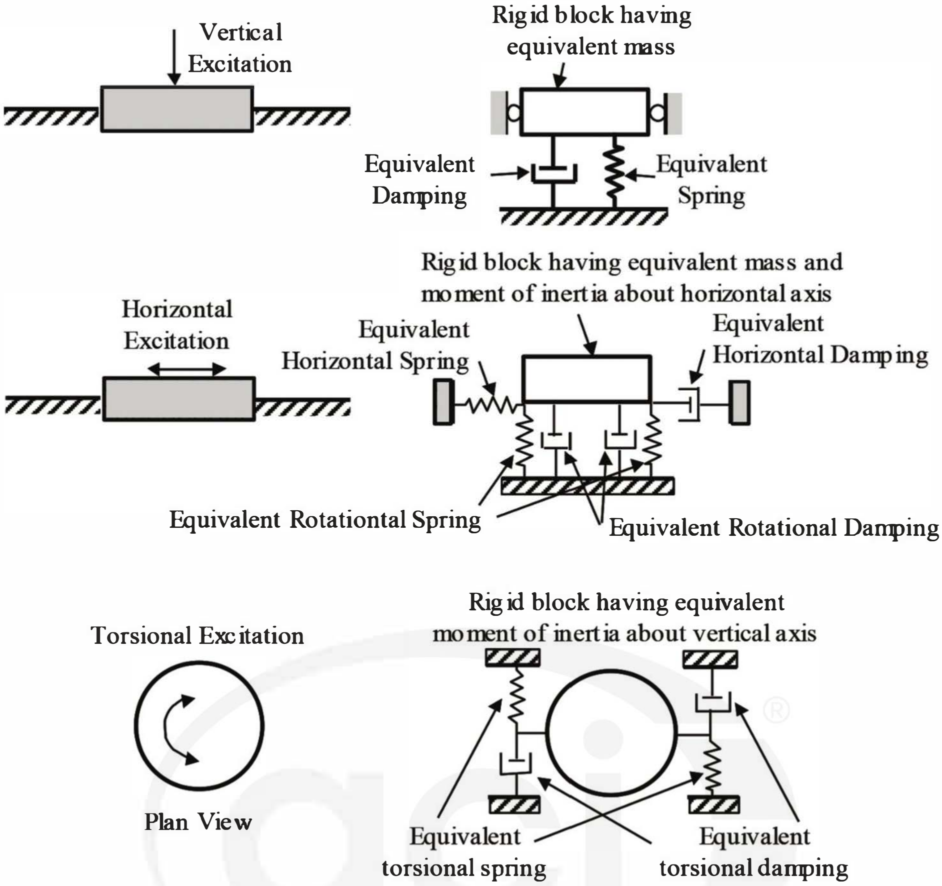


Fig. 5.2a—Lumped system for a foundation subjected to vertical, horizontal, and torsional excitation forces (Richart et al. 1970).

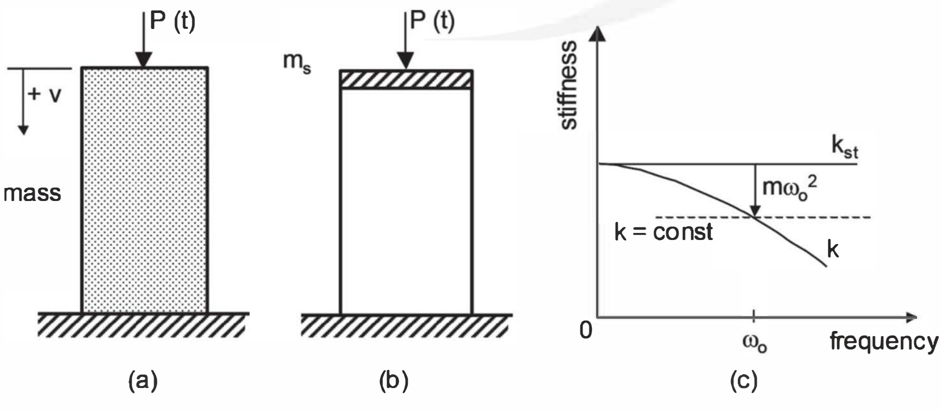


Fig. 5.2b—Effect of mass on dynamic stiffness.

top of the lumped mass is the sum of the elastic force in the column and the inertia force of the mass. If the displacement of this mass varies harmonically

$$u(t) = u_o \cos\omega t \quad (5.2a)$$

Then, the acceleration is

$$\ddot{u} = \frac{d^2u}{dt^2} = -u_o\omega^2 \cos\omega t \quad (5.2b)$$

and the inertia force is

$$m_s\ddot{u} = -m_s u_o \omega^2 \cos\omega t \quad (5.2c)$$

In the absence of damping, the time-dependent relation of the external harmonic force applied with amplitude P_o and frequency ω_o to the displacement u is

$$P(t) = P_o\cos\omega t = k_{st}u_o\cos\omega t - m_s u_o\omega^2\cos\omega t \quad (5.2d)$$

and the amplitudes of force and displacement relate as

$$P_o = k_{st}u_o - m_s u_o\omega^2 \quad (5.2e)$$

The dynamic stiffness, being the constant of proportionality between the applied force and displacement, becomes

$$k = k_{st} - m_s\omega^2 \quad (5.2f)$$

Thus, with vibration of an element having distributed mass, the dynamic stiffness generally varies with frequency. At low frequency, this variation is sometimes close to parabolic, as shown in Fig. 5.2b(c). The column used in this example may represent the column of soil and, thus, a soil deposit always features stiffness parameters that are frequency dependent. The magnitude and character of this frequency effect depend on the size of the foundation geometry, vibration mode, soil layering, and other factors.

The dashpots of Fig. 5.2a are often represented as viscous dampers producing forces proportional to the vibration velocity $\dot{u}$ of the mass. The magnitude of the damper force is

$$F_D = c\dot{u} = c\frac{du}{dt} \quad (5.2g)$$

The damping constant c is defined as the force associated with a unit velocity. During harmonic vibrations, the time-dependent viscous damping force is

$$c\dot{u} = -cu_o\omega\sin\omega t \quad (5.2h)$$

and its peak value is $cu_o\omega$. Equation (5.2h) shows that for a given constant c and displacement amplitude u_o, the amplitude of viscous damping force is proportional to frequency (Fig. 5.2c).

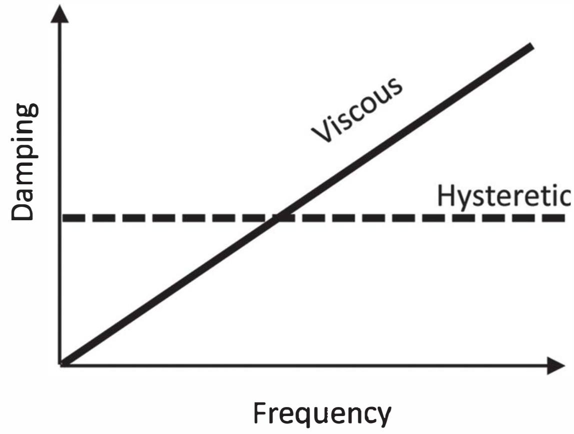


Fig. 5.2c—Comparison of viscous and hysteric damping.

5.3—Calculation of dynamic foundation impedances

The calculation of dynamic impedances is fundamental for the dynamic analysis and design of machine foundations. For instance, as previously discussed, six lumped dynamic impedances—three translational and three rotational dynamic impedances—are needed to model the dynamic response of an infinitely rigid foundation. These impedances are defined and calculated as explained in the following.

5.3.1 *Dynamic impedance definition*—The response of an infinitely rigid foundation to static or dynamic load, P or $P(\omega t)$, arises solely from the deformation, U or $U(\omega t + \phi)$, of the supporting soil. In particular, the static foundation stiffness, $k_{st} = P/U$, is used for modeling the soil-foundation response to static loads. In an analogous manner, the dynamic foundation impedance, $k^*(\omega) = P(\omega t)/U(\omega t + \phi)$, is used for modeling the soil-foundation response to a harmonic (dynamic) load of angular frequency ω (for example, due to rotating machinery). The main difference between the static, k_{st}, and dynamic, $k^*(\omega)$, impedance is that in the static case, the load (P) and displacement (U) occur simultaneously, that is, P and U are in phase, whereas in the dynamic case, the displacement response is frequency dependent and typically lags the loading; that is, there is a phase difference between the two. Therefore, for mathematical convenience, complex notation is used to represent the dynamic impedance of a foundation, given that complex numbers encapsulate magnitude and phase information in a single entity/number. It should be emphasized, however, that the loading and response (vibration amplitude) of a machine foundation are always real; complex numbers are used just for mathematical convenience given that they greatly simplify calculations involving harmonic functions, such the loading acting in machine foundations.

5.3.2 *Dynamic impedance calculation*—The procedure used to calculate the dynamic impedances of a rigid surface foundation can be summarized in the following steps:

a) Model the foundation as massless and infinitely rigid; therefore, only the geometry of the area in contact with the soil is required. The use of a massless foundation is important because it avoids the need for recalculating the dynamic impedance every time the foundation mass changes, which often happens during the design process.

b) Apply a harmonic force or moment of frequency ω to the rigid foundation ($P(t) = P_0e^{i\omega t}$ or $M(t) = M_0e^{i\omega t}$), as shown in Fig. 5.3.2, where $i = \sqrt{-1}$. Such force/moment excites the foundation, which in turn generates stress waves that propagate into the underlying soil. Where the soil is modeled as a viscoelastic material, therefore, the following properties are needed for each soil layer supporting the foundation: thickness (h), modulus of elasticity (E_s), Poisson's ratio (ν), density (ρ), and material damping (β_m).

c) Calculate the vibration response ($U(\omega t + \phi) = U_0e^{i\omega t+i\phi}$ or $\theta(\omega t + \phi) = \theta_0e^{i\omega t+i\phi}$) of the foundation under the applied harmonic excitation. It should be noted that the steady-state vibration amplitude is obtained by keeping track of the

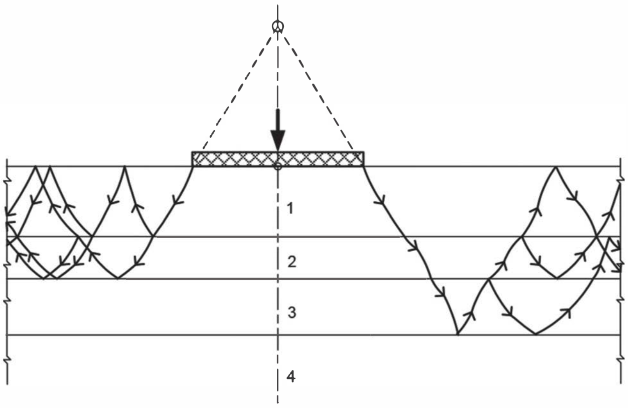


Fig. 5.3.2—Downward and upward wave propagation for surface disk in layered half-space (Wolf and Deeks 2004).

reflections and refractions that take place every time that a stress wave, generated by the foundation vibration, reaches a soil layer boundary. This is achieved by finding the different wave paths shown in Fig. 5.3.2, which can be done analytically or numerically. Note that ϕ represents the phase difference (lag) of the response relative to the applied force or moment.

d) Calculate the translational/rotational dynamic impedance $k^*(\omega)$ as the ratio between the harmonic excitation (force/moment) acting on the foundation and its corresponding vibration amplitude, as shown in Eq. (5.3.2a) and (5.3.2b). It is important to note that $k^*(\omega)$ is a frequency-dependent quantity with a real (Re) and imaginary (Im) part.

$$k^*(\omega) = \frac{P(\omega t)}{U(\omega t + \phi)} = \frac{P_0 e^{i\omega t}}{U_0 e^{i\omega t + i\phi}} = \frac{P_0}{U_0} e^{-i\phi} \quad (5.3.2a)$$

$$k^*(\omega) = \frac{M(\omega t)}{\theta(\omega t + \phi)} = \frac{M_0 e^{i\omega t}}{\theta_0 e^{i\omega t + i\phi}} = \frac{M_0}{\theta_0} e^{-i\phi} \quad (5.3.2b)$$

e) Separate the real and imaginary parts of the dynamic impedance to streamline its use in computer codes used for structural analysis. It is customary to express the complex dynamic impedance as shown in Eq. (5.3.2c). The real and imaginary parts of the dynamic impedance are associated, by analogy to the system described in 5.2, with a dynamic (frequency-dependent) spring and dashpot as shown in Eq. (5.3.2d) and (5.3.2e).

$$k^*(\omega) = k + i\omega C \quad (5.3.2c)$$

$$k(\omega) = \mathrm{Re}(k^*(\omega)) = \frac{P_0}{U_0}\cos\phi \quad (5.3.2d)$$

$$C(\omega) = \frac{\mathrm{Im}(k^*(\omega))}{\omega} = -\frac{P_0}{\omega U_0}\sin\phi \quad (5.3.2e)$$

Equations (5.3.2c) and (5.3.2d) are analogous to Eq. (5.3.2b).

f) Repeat steps (b) to (e) for each frequency of interest until the range of vibration frequencies of the machine is covered.

The aforementioned approach may also be used for calculating the dynamic impedance of a single pile or pile foundation. Because, in this case, the mass and flexibility of each pile are considered, the pile impedance is a function of the pile stiffness and density per unit length. Furthermore, the concept of dynamic impedance can be extended to flexible foundations; however, in this case, it is more natural to define a dynamic impedance matrix. Details regarding the generalization of the dynamic impedance concept can be found in Wolf and Song (1996) and Lysmer et al. (1999).

5.4—Dynamic impedance of soil-supported foundations

The engineer can calculate the dynamic impedance of a soil-supported foundation using the steps presented in 5.3.2 in a finite or boundary element code. Many foundation impedances of practical importance, however, have been published in the literature as closed-form solutions, tables, or charts. Therefore, for application purposes, the engineer can estimate the dynamic impedance, $k^*(\omega)$, of a soil-supported foundation by one or a combination of the following approaches:

a) *Frequency-dependent equations*—Veletsos and Nair (1974), Veletsos and Verbic (1973), and Veletsos and Wei (1971) have developed appropriate equations that represent the impedance to motion offered by uniform soil conditions. They developed these formulations using an assumption of a uniform elastic or viscoelastic half-space and the related motion of a rigid or flexible foundation on this half-space. Motions can be translational or rotational. While the calculations can be manually tedious, computer implementation provides acceptable efficiency. Some of the simpler equations are presented in 5.4.1.2 of this report. More extensive equations are presented in Gazetas (1991) and Sieffert and Cevaer (1992).

b) *Constant approximation*—If the frequency range of interest is not very wide, an often-satisfactory technique is to replace the variable dynamic stiffness by the constant representative of the stiffness in the vicinity of the dominant frequency (Fig. 5.2b(c)). Another constant approximation can be to add an effective or fictitious in-phase mass of the supporting soil medium to the vibrating lumped mass of the machine and foundation and to consider stiffness to be constant and equal to the static stiffness. Nevertheless, the variation of dynamic stiffness with frequency can be represented by a parabola only in some cases, and the added mass is not the same for all vibration modes.

c) *Computer-based numerical analysis*—For complex geometry, flexible foundations, or variable soil conditions, the recommended approach is to obtain the stiffness and damping of foundations using dynamic analysis of a three- or two-dimensional continuum representing the soil medium. The continuum is modeled as an elastic or viscoelastic half-space. The half-space can be homogeneous or nonhomoge-

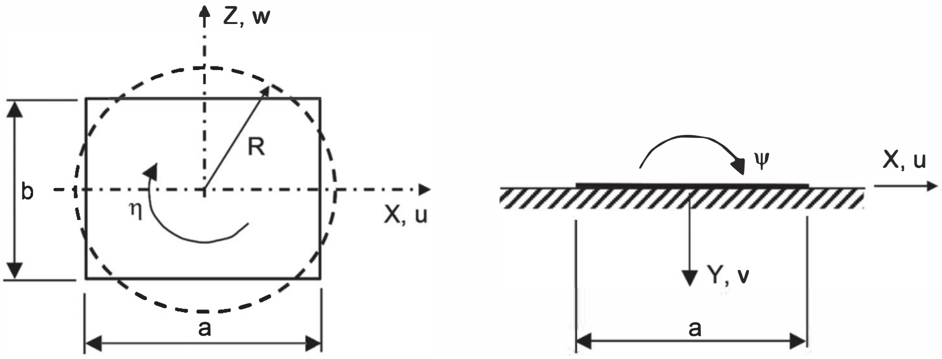


Fig. 5.4—Notation for calculation of equivalent radii for rectangular bases.

neous (layered) and isotropic or anisotropic. The governing equations are solved analytically or by means of numerical methods such as the finite element method. The major advantages of this elastic half-space technique are that it accounts for energy dissipation through elastic waves (geometric damping, also known as radiation damping), provides for systematic analysis, and uses soil properties defined by constants that can be established by independent experiments. Computer programs are very useful in this regard.

Commonly, the analysis of dynamically loaded foundations is performed considering the rectangular foundation as if it were a circular foundation with equivalent properties (Fig. 5.4). Bowles (1996) and Gazetas (1991) provide alternates to this approach. The equivalent circular foundation is not the same for all directions of motion. For the three translational directions, the equivalent circular foundation is determined based on an area equivalent to the rectangular foundation; thus, these three share a common equivalent radius *R*. For the three rotational directions, three unique radii are determined that have moments of inertia equivalent to their rectangular counterparts. These relationships are reflected in the following equations.

Translation:

$$R = \sqrt{\frac{ab}{\pi}} \tag{5.4a}$$

Rocking (about an axis parallel to *b*):

$$R_{\psi b} = \sqrt[4]{\frac{a^3 b}{3\pi}} \tag{5.4b}$$

Rocking (about an axis parallel to *a*):

$$R_{\psi a} = \sqrt[4]{\frac{b^3 a}{3\pi}} \tag{5.4c}$$

Torsion:

$$R_{\eta} = \sqrt[4]{\frac{ab(a^2 + b^2)}{6\pi}} \tag{5.4d}$$

The equivalent radius works very well for square foundations and rectangular foundations with aspect ratios *a*/*b* of up to 2. With *a*/*b* above 2, the accuracy decreases. For long foundations, the assumption of an infinite strip foundation may be more appropriate (Gazetas 1983). Alternatively, the formulas and charts provided by Dobry and Gazetas (1986) and Gazetas (1991) can be used for the analysis of rigid surface foundations of arbitrary shape.

These dynamics problems are addressed either with real domain mathematics or with complex domain mathematics, as illustrated in 5.2 and 5.3.2, respectively. The real domain solution is more easily understood and is typified by the damped stiffness models of Richart and Whitman (1967) and Richart et al. (1970), in which the stiffness and damping are represented as constants. The complex domain impedance is easier to describe mathematically and is applied in the impedance models of Veletsos and Nair (1974), Veletsos and Verbic (1973), and Veletsos and Wei (1971). Equation (5.4e) characterizes the relationship between impedance models and damped stiffness models.

$$k_i^* = k_i + i\omega c_i \tag{5.4e}$$

Another common approach is to calculate impedance parameters based on a dimensionless frequency a_o, first introduced by Reissner (1936), and calculated as

$$a_o = \omega R / V_s = \omega R \sqrt{\rho / G} \tag{5.4f}$$

In the most common circumstances, the soil and foundation are moving at the same frequency the machine operates at. Thus, ω is commonly equal to ω_o. In unusual cases, such as when motions result from a multiple harmonic excitation of the equipment speed, from blade passage effects, from multiple speed equipment combinations, and from some poor bearing conditions, the motion may develop at a speed different from the equipment speed. These situations are more commonly found during in-place problem investigations rather than initial design applications. In those cases, the correct frequency to consider is determined by in-place field measurements.

5.4.1 *Uniform soil conditions*—In many instances, engineers approximate or assume that the soil conditions are uniform throughout the depth of interest for the foundation being designed, which permits the use of a homogeneous half-space model. This assumption, in some cases, is reasonable. In other cases, it is accepted for lack of any better model and because the scope of the foundation design does not warrant more sophisticated techniques. However, significant error is introduced when this assumption is used for the analysis of foundations supported by a layer resting on a rigid substratum or for foundations on layered soils with differing properties.

5.4.1.1 *Richart-Whitman models*—Engineers frequently use a lumped parameter model (Richart and Whitman 1967), which is considered suitable for uniform soil conditions. This model represents the stiffness, damping, and mass for each mode as single, lumped parameters. In the norm, these parameters are treated as frequency-independent. For the vertical direction, the stiffness and damping equations are expressly validated for the range of dimensionless frequencies from 0 to 1.0 ($0 < a_o < 1.0$) (Richart et al. 1970). The other directions are similarly limited, as the stiffness parameters are actually static stiffness values.

The mass of these models is determined solely as the translational mass and rotational mass moment of inertia for the appropriate directions. No effective soil mass is included in the representation. The mass and damping ratios recommended by Richart et al. (1970) for infinitely rigid, circular foundations are given by the following equations. Lysmer and Richart (1966) first proposed this formulation for the vertical translation case.

Vertical:

$$B_v = \frac{(1-\nu)}{4}\frac{m}{\rho R^3} \quad (5.4.1.1a)$$

and

$$D_v = \frac{0.425}{\sqrt{B_v}} \quad (5.4.1.1b)$$

Horizontal:

$$B_u = \frac{(7-8\nu)}{32(1-\nu)}\frac{m}{\rho R^3} \quad (5.4.1.1c)$$

and

$$D_u = \frac{0.288}{\sqrt{B_u}} \quad (5.4.1.1d)$$

Rocking:

$$B_\psi = \frac{3(1-\nu)}{8}\frac{I_\psi}{\rho R^5_\psi} \quad (5.4.1.1e)$$

and

$$D_\psi = \frac{0.15}{(1+B_\psi)\sqrt{B_\psi}} \quad (5.4.1.1f)$$

Torsion:

$$B_\eta = \frac{I_\eta}{\rho R^5_\eta} \quad (5.4.1.1g)$$

and

$$D_\eta = \frac{0.50}{(1+2B_\eta)} \quad (5.4.1.1h)$$

The lumped parameter models generally recognize two alternate representations for the lumped stiffness terms. These equations are based on the theory of elasticity for elastic half-spaces with infinitely rigid foundations, except for horizontal motions, which use a slightly different rigidity assumption. For a rigid circular foundation, the applicable formulas are given in Eq. (5.4.1.1i) to (5.4.1.1l). For a rectangular foundation, the equivalent radii of Eq. (5.4a) to (5.4d) can be used in the calculations if $a/b < 2.0$. Alternatively, Eq. (5.4.1.1m) to (5.4.1.1o) can be used directly for rectangular foundations of plan dimensions a by b. In these equations, the β_i values vary with the aspect ratio of the rectangle. The β_v value for vertical vibration ranges from approximately 2.1 to 2.8, β_u for the horizontal vibration ranges from approximately 0.95 to 1.2, and β_ψ for the rocking vibration varies from approximately 0.35 to 1.25. Specific relationships can be found in a variety of sources (Richart et al. 1970; Arya et al. 1979; Richart and Whitman 1967), usually in a graphical form, as shown in Fig. 5.4.1.1. The original source materials contain the mathematical relationships. There is little difference between using either the circular or the rectangular formulations for these stiffness terms.

Vertical:

$$k_v = \frac{4}{(1-\nu)}GR \quad (5.4.1.1i)$$

Horizontal:

$$k_u = \frac{32(1-\nu)}{(7-8\nu)}GR \quad (5.4.1.1j)$$

Rocking:

$$k_\psi = \frac{8}{3(1-\nu)}GR_\psi^{\,3} \quad (5.4.1.1k)$$

Torsion:

$$k_\eta = \frac{16}{3} G R_\eta^{\,3} \quad (5.4.1.1l)$$

Vertical:

$$k_v = \frac{G}{(1-v)} \beta_v \sqrt{ab} \quad (5.4.1.1m)$$

Horizontal:

$$k_u = 2(1+v) G \beta_u \sqrt{ab} \quad (5.4.1.1n)$$

Rocking (about an axis parallel to b):

$$k_\psi = \frac{G}{(1-v)} \beta_\psi b a^2 \quad (5.4.1.1o)$$

The damping constants can be determined from the calculated damping ratio, system mass, and stiffness as

$$c_i = 2 D_i \sqrt{k_i m} \quad (5.4.1.1p)$$

or

$$c_i = 2 D_i \sqrt{k_i I_i} \quad (5.4.1.1q)$$

5.4.1.2 *Veletsos models*—For the impedance functions of infinitely rigid circular foundations resting on the surface of a viscoelastic half-space, Veletsos and Verbic (1973) determined the analytical expressions for the dynamic impedance as a function of frequency, Poisson's ratio, and internal material damping. Neglecting the internal material damping, the relationships for an infinitely rigid circular foundation are

Horizontal impedance:

$$k_u^* = \frac{8GR}{2-v}[1 + i a_o \alpha_1] \quad (5.4.1.2a)$$

Rocking impedance:

$$k_\psi^* = \frac{8GR_\psi^{\,3}}{3(1-v)}[(1 - \chi_\psi - \beta_3 a_o^2) + i a_o \psi_\psi] \quad (5.4.1.2b)$$

where $\chi_\psi = \dfrac{\beta_1 \cdot (\beta_2 a_o)^2}{1 + (\beta_2 a_o)^2}$ and $\psi_\psi = \dfrac{\beta_1 \beta_2 \cdot (\beta_2 a_o)^2}{1 + (\beta_2 a_o)^2}$

Vertical impedance:

$$k_v^* = \frac{4GR}{(1-v)}[(1 - \chi_v - \gamma_3 a_o^2) + i a_o(\gamma_4 + \psi_v)] \quad (5.4.1.2c)$$

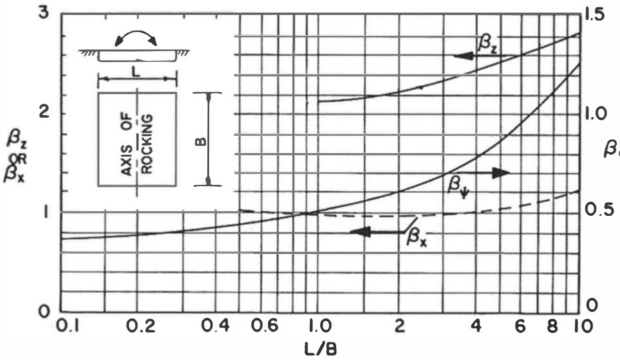


Fig. 5.4.1.1—Rectangular footing coefficients (Richart et al. 1970).

where $\chi_v = \dfrac{\gamma_1 \cdot (\gamma_2 a_o)^2}{1 + (\gamma_2 a_o)^2}$ and $\psi_v = \dfrac{\gamma_1 \gamma_2 \cdot (\gamma_2 a_o)^2}{1 + (\gamma_2 a_o)^2}$

Torsional impedance:

$$k_\eta^* = \frac{16 G R_\eta^3}{3}[A + i a_o B] \quad (5.4.1.2d)$$

where $A = 1 - \dfrac{b_1 \cdot (b_2 a_o)^2}{1 + (b_2 a_o)^2}$ and $B = \dfrac{b_1 b_2 \cdot (b_2 a_o)^2}{1 + (b_2 a_o)^2}$

where j = 1 to 4 as appropriate and b_1 and b_2 = 0.425 and 0.687, respectively (Veletsos and Nair 1974).

The leading terms of each of these impedance equations are the static stiffness of the foundation for vibration in that direction. For three cases (vertical, rocking, and torsional motion), these terms are the same as presented in the Richart-Whitman (Richart and Whitman 1967) lumped parameter model. For the horizontal motion, there is a slight difference due to assumptions about the foundation rigidity. The practicality of the difference is small, especially toward the higher range of Poisson's ratio values. These equations may also be expressed as

Horizontal impedance:

$$k_u^* = GR(C_{u1} + i a_o C_{u2}) \quad (5.4.1.2e)$$

Rocking impedance:

$$k_\psi^* = GR_\psi^{\,3}(C_{\psi 1} + i a_o C_{\psi 2}) \quad (5.4.1.2f)$$

Vertical impedance:

$$k_v^* = GR(C_{v1} + i a_o C_{v2}) \quad (5.4.1.2g)$$

Torsional impedance:

$$k_\eta^* = GR_\eta^{\,3}(C_{\eta 1} + i a_o C_{\eta 2}) \quad (5.4.1.2h)$$

Constant approximations of C_{i1} and C_{i2} are given in Table 5.4.1.2 for two broad classes of soils (cohesive and granular) and for dimensionless frequency values a_o less than 2.0. The

Table 5.4.1.2—Stiffness and damping parameters (*D* = 0)

Motion	Soil	Side layer		Half space	
Horizontal	Cohesive	$S_{u1} = 4.1$	$S_{u2} = 10.6$	$C_{u1} = 5.1$	$C_{u2} = 3.2$
	Granular	$S_{u1} = 4.0$	$S_{u2} = 9.1$	$C_{u1} = 4.7$	$C_{u2} = 2.8$
Rocking	Cohesive	$S_{\psi 1} = 2.5$	$S_{\psi 2} = 1.8$	$C_{\psi 1} = 4.3$	$C_{\psi 2} = 0.7$
	Granular	$S_{\psi 1} = 2.5$	$S_{\psi 2} = 1.8$	$C_{\psi 1} = 3.3$	$C_{\psi 2} = 0.5$
Torsion	Cohesive	$S_{\eta 1} = 10.2$	$S_{\eta 2} = 5.4$	$C_{\eta 1} = 4.3$	$C_{\eta 2} = 0.7$
	Granular	$S_{\eta 1} = 10.2$	$S_{\eta 2} = 5.4$	$C_{\eta 1} = 4.3$	$C_{\eta 2} = 0.7$
Vertical	Cohesive	$S_{v1} = 2.7$	$S_{v2} = 6.7$	$C_{v1} = 7.5$	$C_{v2} = 6.8$
	Granular	$S_{v1} = 2.7$	$S_{v2} = 6.7$	$C_{v1} = 5.2$	$C_{v2} = 5.0$

Note: *S* values are valid for $0.5 < a_o < 1.5$; *C* values for valid for $a_o < 2.0$.

polynomial expansions given in Eq. (5.4.1.2a) to (5.4.1.2d) cover a wide range of dimensionless frequencies. While the practical application of the closed-form solution presented in this section and in 5.4.1.1 is rather limited because real foundations are not infinitely rigid or rest on a homogeneous half-space, closed-form solutions are important because they can be used to validate numerical impedance calculations. Therefore, the engineer should become familiar with these formulations and use them to judge the results from more complex approaches. For this purpose, the compilation of dynamic foundation impedances provided by Sieffert and Cevaer (1992) is particularly useful.

5.4.1.3 *Other methods*—Engineers have used finite element models as another approach to obtain stiffness and damping relationships. Rather than theoretically assessing the behavior of an elastic continuum, parametric studies with a finite element model determine the impedance relationships. Given all the approximations involved, the agreement between the solutions—the half-space theory and the finite element modeling—is very good. The exception is the rocking stiffness k_ψ for which researchers have calculated substantially larger values by finite element solutions (Karabalis and Beskos 1985; Kobayashi and Nishimura 1983; Wolf and Darbre 1984). Additionally, empirical expressions for the static stiffness of circular foundations embedded into a homogeneous soil layer can be found in Elsabee and Morray (1977) and Kausel and Ushijima (1979).

5.4.2 *Adjustments to theoretical values*—Damping in a soil-foundation system consists of two components: geometric damping and material damping. The geometric damping represents a measure of the energy radiated away from the immediate region of the foundation by the soil. Material damping measures the energy loss as a result of soil hysteresis effects.

Sections 5.4.1.1 and 5.4.1.2 present equations to evaluate geometric damping for various foundation geometries and different types of soils using elastic half-space theories. Both experience and experimental results show that damping values for large foundations undergoing small vibration amplitudes are typically less than those predicted analytically (EPRI 1980; Novak 1970). This discrepancy is attributed to the presence of soil layers that reflect waves back to the vibrating foundation. Various sources recommend reduced geometric soil damping values (EPRI 1980; Novak 1970; Gazetas 1983). Specific recommendations vary with the type of application. The engineer needs to select proper soil damping values and limits based on the specific application. For example, EPRI (1980) recommends the soil damping ratio for use in the design of power plant fan foundations should not exceed 20 percent for horizontal motion, 50 percent for vertical motion, 10 percent for transverse rocking motion, and 15 percent for axial and torsional motions. DIN 4024-2 recommends that the soil damping ratios used in vibration analysis of rigid block foundations should not exceed 25 percent. Novak (1970) recommends reducing the analytically determined geometric damping ratios (from elastic half-space models) by 50 percent for a dynamic analysis of the foundation. Therefore, the engineer should generally be conservative in the application of analytically determined damping values.

Although material damping is often neglected, as presented in 5.4.1, that assumption often leads to overestimating the first resonant amplitude of the coupled translation/rocking response of surface footings by several hundred percent because very limited geometric damping develops during rocking. This overestimation can be reduced by the inclusion of material damping (5.4.4).

Torsional response is difficult to predict because of slippage. For surface foundations, slippage reduces stiffness and increases damping; for embedded foundations, slippage reduces damping. The inclusion of the weakened zone around the footing may improve the agreement between the theory and the experiments. This typically requires more complete computer-based numerical analysis.

Another correction of the half-space theory may be required if the soil deposit is a shallow layer resting on a rigid substratum. In such cases, the stiffness increases and geometric damping decreases and can even vanish if the frequency of interest (for example, the excitation frequency) is lower than the first natural frequency of the soil layer (Kausel and Ushijima 1979). For a homogeneous layer of thickness *H* with soil shear wave velocity V_s, the first natural frequencies of the soil deposit are

Horizontal direction:

$$\omega_{su} = \frac{\pi V_s}{2H} \qquad (5.4.2a)$$

Vertical direction:

$$\omega_{sv} = \frac{\pi V_s}{2H}\sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad (5.4.2b)$$

At excitation frequencies ω_o lower than ω_{su} and ω_{sv}, only material damping remains because no progressive wave occurs to generate geometric damping in the absence of material damping, and only a very weak progressive wave occurs in the presence of material damping. The damping parameters generated by the material alone to be used for excitation frequencies ω_o lower than ω_{su} and ω_{sv} are

$$c_u = 2\beta_m \times k_u/\omega_o \quad (5.4.2c)$$

$$c_v = 2\beta_m \times k_v/\omega_o \quad (5.4.2d)$$

where β_m is the material damping ratio.

In the impedance models, the imaginary terms of the horizontal (k_u^*) and vertical (k_v^*) impedance become $+2\beta_m i$ within the brackets of Eq. (5.4.1.2a) and (5.4.1.2c). This correction is most important for the vertical and horizontal directions, in which the geometric damping of the half-space is high but is minimal for the shallow layer. This correction is not an absolute breakpoint based on the calculated layer frequency and the excitation frequency. Therefore, the recommended approach is to use computer-based numerical solution techniques that reasonably represent the loss of geometric damping. If this is not possible, the engineer should apply judgment to decrease the geometric damping for shallow layer sites, as well as the aforementioned recommendations.

5.4.3 *Embedment effects*—Most footings do not rest on the surface of the soil but are partly embedded. Studies have shown that embedment increases both stiffness and damping, but the increase in damping is more significant (Kausel et al. 1977; Tyapin 1991).

Overall, embedment effects are often overestimated because soil stiffness (shear modulus) diminishes toward the soil surface due to diminishing confining pressure. When the embedment consists of backfill, the specific characteristics of the backfill should always be considered in evaluating soil stiffness. The lack of confining pressure at the surface often leads to separation of the soil from the foundation and to the creation of a gap, as indicated in Fig. 5.4.3, which significantly reduces the effectiveness of embedment. To find an approximate correction for this effect, the engineer should consider an effective embedment depth less than the true embedment.

In determining the stiffness of embedded foundations, one approximate but versatile approach is to assume that the soil reactions acting on the base can be taken as equal to those of a surface foundation (half-space) and the reactions acting on the footing sides as equal to those of an independent layer overlying the half-space (Fig. 5.4.3). The evaluation of the reactions of a layer are simplified and calculated using the assumption of plane strain. This means that these reactions are taken as equal to those of a rigid, infinitely long, massless cylinder undergoing a uniform motion in an infinite homogeneous medium.

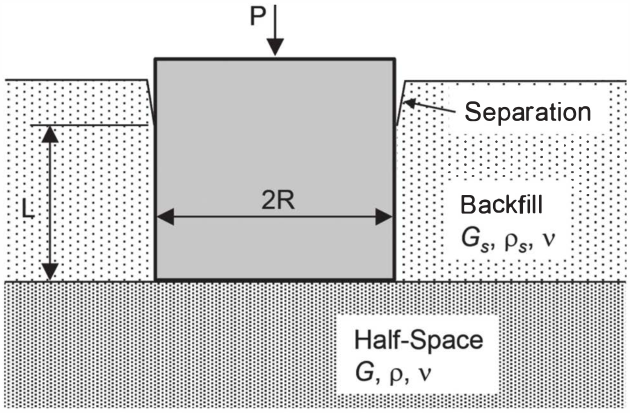


Fig. 5.4.3—Schematic of embedded foundation.

The plane strain approach to the side reactions has many advantages: it accounts for energy radiation through wave propagation, leads to closed-form solutions, and allows for the variation of the soil properties with depth. It can also allow for the slippage zone around the footing. Also, the approach is simple and makes it possible to use the solutions of surface footings because the effect of the independent side layer actually represents an approximate correction of the half-space solutions for the embedment effect. This approach works quite well, and its accuracy increases with increasing frequency. Alternatively, the formulas and charts provided by Gazetas (1991) can be used if more accurate expressions are required.

Equations (5.4.3a) to (5.4.3d) describe the side resistance of the embedded cylinder, analogous to the surface disk, using complex, frequency-dependent impedance

Horizontal direction:

$$k_{eu}^* = G_s l[S_{u1} + i \cdot a_o \cdot S_{u2}] \quad (5.4.3a)$$

Vertical impedance:

$$k_{ev}^* = G_s l[S_{v1} + i \cdot a_o \cdot S_{v2}] \quad (5.4.3b)$$

Rocking impedance:

$$k_{e\psi}^* = G_s R_\psi^2 l[S_{\psi 1} + i \cdot a_o \cdot S_{\psi 2}] \quad (5.4.3c)$$

Torsional impedance:

$$k_{e\eta}^* = G_s R_\eta^2 l[S_{\eta 1} + i \cdot a_o \cdot S_{\eta 2}] \quad (5.4.3d)$$

In these expressions, the dynamic shear modulus G_s is that of the side layer that may represent the backfill. The dimensionless parameters S_{i1} and S_{i2} relate to the real stiffness and the damping (out-of-phase component of the impedance), respectively. These parameters depend on the dimensionless frequency a_o (Eq. (5.4f)) applicable for the layer of embedment material. Poisson's ratio affects only the horizontal impedance generated by the footing embedment, not the impedance in other directions. In complete form, these S_i parameters also depend on material damping of the side layer soils.

The mathematical expressions for the parameters S_{i1} and S_{i2} can be found in Novak et al. (1978) and Novak and Sheta (1980). These parameters are frequency-dependent; nevertheless, given all the approximations involved in the modeling dynamic soil behavior, it is often sufficient to select suitable constant values to represent the parameters, at least over limited frequency range of interest. Table 5.4.1.2 shows constant values for cohesive soils and granular soils with Poisson's ratio of 0.4 and 0.25, respectively. The values correspond to dimensionless frequencies between 0.5 and 1.5, which are typical of many machine foundations. If a large frequency range is important, parameter S should be considered as frequency-dependent and calculated from the expression of impedance functions given in Novak et al. (1978) and Novak and Sheta (1980). Material damping is not included in Table 5.4.1.2 but can be accounted for by using Eq. (5.4.4b).

The engineer can approximate the complex stiffness of embedded foundations by adding the stiffness generated by the footing sides to that generated in the base. In some cases, consideration of the difference in location of the embedment impedance and the basic soil impedance may be included in the analysis. For vertical translation and torsion, the total stiffness and damping results in simple addition of the two values. For horizontal translation and rocking, coupling between the two motions should be considered.

5.4.4 *Material damping*—Material damping can be incorporated into the stiffness and damping of the footing in several ways. The most direct way is to introduce the complex shear modulus into the governing equations of the soil medium at the beginning of the analysis and to carry out the entire solution with material damping included.

Another way is to carry out the purely elastic solution and then introduce material damping into the results by applying the correspondence principle of viscoelasticity. With steady-state oscillations considered in the derivation of footing stiffness, the application of the correspondence principle consists of replacing the real shear modulus G by the complex shear modulus G^*.

This replacement should be done consistently wherever G occurs in the elastic solution. This includes the shear wave velocity and the dimensionless frequency (Eq. (5.4f)), which consequently become complex. Therefore, all functions that depend on a_o are complex as well. The substitution of G^* can be done if analytical expressions for the impedance k_i^* or constants k_i and c_i are available from the elastic solution. With the material damping included, the parameters have the same meaning as before, but also depend on the material damping.

The aforementioned procedures for the inclusion of material damping into an elastic solution are accurate but not always convenient. When the elastic solution is obtained using a numerical method, the impedance functions are obtained in a digital or graphical form, and analytical expressions are not available. In such cases, an approximate approach often used to account for material damping multiplies the complex impedance, evaluated without regard to material damping, by the complex factor $(1 + i2\beta_m)$ to determine an adjusted complex impedance

$$\begin{aligned} k_i^*(\text{adj}) &= (k_i + i\omega c_i) \times (1 + 2i\beta_m) \\ &= (k_i - 2\beta_m\,\omega c_i) + i\omega\,(c_i + 2\beta_m\,k_i/\omega) \end{aligned} \quad (5.4.4a)$$

Recognizing the stiffness as the real part of the impedance and the damping as the imaginary term of the impedance, the adjusted stiffness and damping terms considering material damping become

$$k_i(\text{adj}) = k_i - 2\beta_m\omega c_i \quad (5.4.4b)$$

$$c_i(\text{adj}) = c_i + 2\beta_m k_i/\omega \quad (5.4.4c)$$

where k_i and c_i are calculated assuming perfect elasticity, and c_i includes only geometric damping. Studies indicate that this approximate approach gives sufficient accuracy at low dimensionless frequencies, but the accuracy deteriorates with increasing frequency. Equations (5.4.4b) and (5.4.4c) show that material damping reduces stiffness in addition to increasing damping.

As another approach, variations of Eq. (5.4.1.2a) to (5.4.1.2d) are available, which include material damping directly as a distinct parameter (Veletsos and Verbic 1973; Veletsos and Nair 1974).

Finally, some engineers simply add the material damping to the geometric damping otherwise determined by the preceding equations. This approach is more commonly used with the Richart-Whitman (Richart and Whitman 1967) formulations and does not alter the stiffness. In such circumstances, broad judgments are often applied at the same time so that if the geometric damping is large, the material damping may be neglected. Similarly, material damping may be included only in those cases where excessive resonance amplification is expected. This simple additive approach is generally recognized as the least accurate of the possible methods.

5.5—Dynamic impedance of pile foundations

Stiffness and damping of piles are affected by interaction of the piles with the surrounding soil. The soil-pile interaction under dynamic loading modifies the pile stiffness, making it frequency dependent. As with shallow foundations, this interaction also generates geometric damping. In groups of closely spaced piles, the character of dynamic stiffness and damping is further complicated by interaction between individual piles known as pile-soil-pile interaction or group effect.

Therefore, recent approaches for determining stiffness and damping of piles consider soil-pile interaction in terms of continuum mechanics and account for propagation of elastic waves. The solutions to the problem are based on various approaches such as continuum mechanics methods, lumped mass models, the finite element model, and the boundary integral method (Fig. 5.5).

In most cases, the impedance of a pile foundation is determined considering only the impedance of the piles. Because piles are typically used due to poor surface layer soils, the effects of the soils directly under the cap are often neglected; a settlement gap is assumed to develop. Similarly, the effects of embedment are often neglected. If circumstances indicate that the embedment effect may be significant, the procedures outlined in 5.4.3 are often applied.

The basic approach toward pile analysis is to first evaluate the impedance parameters of a single pile. Once these parameters (stiffness and damping) are established for the single pile, the group effects are determined. Other approaches, such as finite element analysis, may model and consider both effects simultaneously.

5.5.1 *Single piles*—Dynamic behavior of embedded piles depends on excitation frequency and the material and geometric properties of both the pile and soil. The pile is described by its length, bending and axial stiffness, tip/end conditions, mass, and batter (inclination from the vertical). Soil behavior depends on soil properties and the soil's variation with depth (layering).

Dynamic response of a pile-supported foundation depends on the dynamic stiffness and damping of the piles. These properties for a single pile can be described in terms of either impedance or dynamic stiffness and equivalent viscous damping. As previously established, these are related as

$$k_i^* = k_i + i\omega c_i \tag{5.5.1a}$$

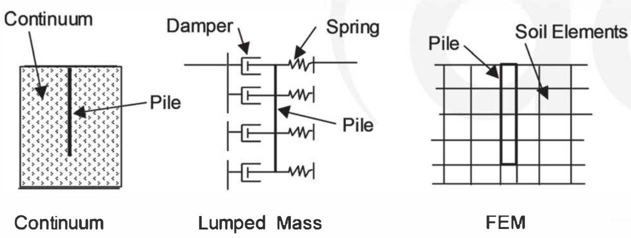


Fig. 5.5—Mathematical models used for dynamic analysis of piles.

The engineer can determine the constants experimentally or theoretically. The theoretical approach is commonly used because experiments, though very useful, are difficult to generalize. In the theoretical approach, the dynamic stiffness is obtained by calculating the forces needed to produce a unit amplitude vibration of the pile head in the prescribed direction as shown in Fig. 5.5.1a and discussed in 5.3. For a single pile, the impedance at the pile head can be determined from the following equations

Vertical translation:

$$k_{vj} = \frac{E_p A_p}{r_o} f_{v1} \tag{5.5.1b}$$

and

$$c_{vj} = \frac{E_p A_p}{V_s} f_{v2} \tag{5.5.1c}$$

Horizontal translation:

$$k_{uj} = \frac{E_p I_p}{r_o^3} f_{u1} \tag{5.5.1d}$$

and

$$c_{uj} = \frac{E_p A_p}{r_o^2 V_s} f_{u2} \tag{5.5.1e}$$

Rotation of the pile head in the vertical plane:

$$k_{\psi j} = \frac{E_p I_p}{r_o} f_{\psi 1} \tag{5.5.1f}$$

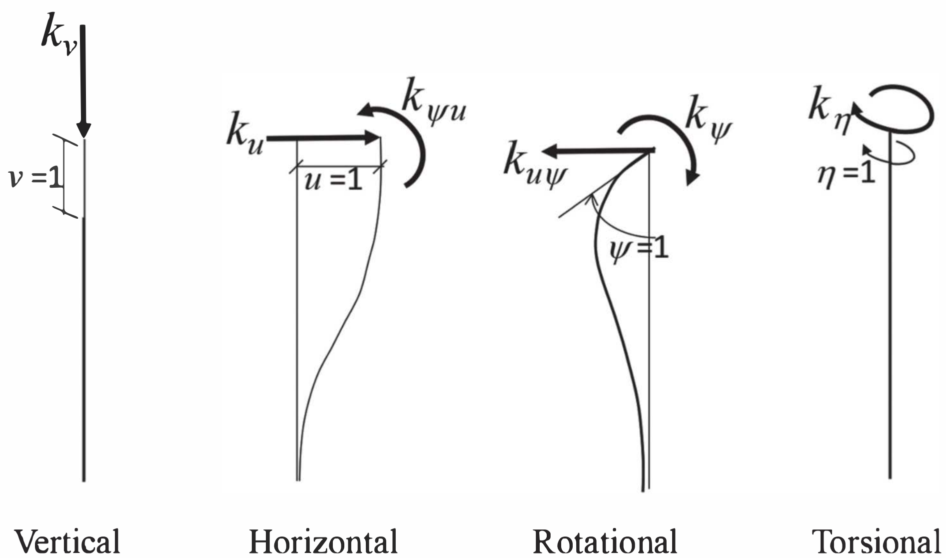


Fig. 5.5.1a—Generation of pile stiffness in individual directions.

Table 5.5.1—Stiffness and damping parameters for Eq. (5.5.1a) to (5.5.1k) for l_p/r_o > 25 (Novak 1974)

			Stiffness parameters			Damping parameters		
ν	ρ/ρ_p	V_s/v_c	$f_{7,1}$	$f_{9,1}$	$f_{11,1}$	$f_{7,2}$	$f_{9,2}$	$f_{11,2}$
0.4	0.7 (Concrete)	0.01	0.202	–0.0194	0.0036	0.139	–0.0280	0.0084
		0.02	0.285	–0.0388	0.0100	0.200	–0.0566	0.0238
		0.03	0.349	–0.0582	0.0185	0.243	–0.0848	0.0438
		0.04	0.403	–0.0776	0.0284	0.281	–0.1130	0.0674
		0.05	0.450	–0.0970	0.0397	0.314	–0.1410	0.0942
0.25	0.7 (Concrete)	0.01	0.195	–0.0181	0.0032	0.135	–0.0262	0.0076
		0.02	0.275	–0.0362	0.0090	0.192	–0.0529	0.0215
		0.03	0.337	–0.0543	0.0166	0.235	–0.0793	0.0395
		0.04	0.389	–0.0724	0.0256	0.272	–0.1057	0.0608
		0.05	0.437	–0.0905	0.0358	0.304	–0.1321	0.0850

Note: $f_{7,1} = f_{\psi 1}$; $f_{7,2} = f_{\psi 2}$; $f_{9,1} = f_{u\psi 1}$; $f_{9,2} = f_{u\psi 2}$; $f_{11,1} = f_{u1}$; and $f_{11,2} = f_{u2}$.

and

$$c_{\psi j} = \frac{E_p I_p}{V_s} f_{\psi 2} \tag{5.5.1g}$$

Coupling between horizontal translation and rotation:

$$k_{u\psi j} = \frac{E_p Ip}{r_0^2} f_{u\psi 1} \tag{5.5.1h}$$

and

$$c_{u\psi j} = \frac{E_p I_p}{r_o V_s} f_{u\psi 2} \tag{5.5.1i}$$

Torsion:

$$k_{\eta j} = \frac{G_p J}{r_o} f_{\eta 1} \tag{5.5.1j}$$

and

$$c_{\eta j} = \frac{G_p J}{V_s} f_{\eta 2} \tag{5.5.1k}$$

Graphical or tabular compilations of the f_{i1} and f_{i2} functions are presented in a variety of sources (Novak 1974, 1977; Kuhlmeyer 1979a,b) and are included in some software packages. Table 5.5.1 and Fig. 5.5.1b show these functions for concrete piles. Note that these functions are generally dependent on many parameters, and the original source (Novak 1974) should be consulted for further information. If the pile heads are pinned into the foundation block, then $k_\psi = k_{u\psi} = k_\eta = 0$ and $c_\psi = c_{u\psi} = c_\eta = 0$ in the previous formulas, and k_u should be evaluated for pinned head piles. The vertical constants labeled v are the same for the fixed

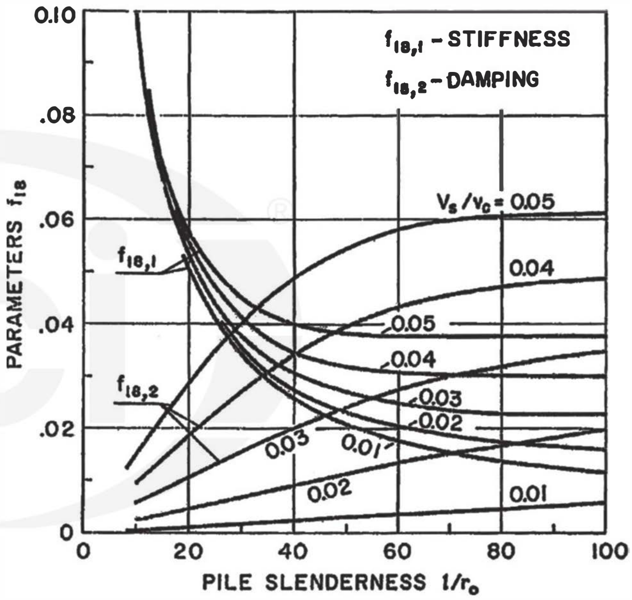


Fig. 5.5.1b—Stiffness and damping parameters for Eq. (5.5.1b) and (5.5.1c) (Novak 1974). Note: $f_{18,1} = f_{v1}$ and $f_{18,2} = f_{v2}$. Also, v_c is the compression wave velocity of the pile (refer to section 5.5.1).

and pinned heads. The rotational parameters ($f_{\psi 1}$, $f_{\psi 2}$, $f_{u\psi 1}$, $f_{u\psi 2}$, $f_{\eta 1}$, and $f_{\eta 2}$) are applicable only if the pile is assumed or designed to be rotationally fixed to the pile cap. In general, the f_{i1} and f_{i2} functions depend on the following dimensionless parameters:

a) Dimensionless frequency $a_o = \omega r_o/V_s$ (note that this value is calculated using the pile radius and typically is much smaller than the a_o calculated for a complete soil supported foundation)

b) Relative stiffness of the soil to the pile, which can be described either by the modulus ratio G/E_p or the velocity ratio V_s/V_c in which V_c is the compression wave velocity of the pile equal to $\sqrt{E_p/\rho_p}$ with ρ_p equal to the pile mass density

c) The mass density ratio ρ/ρ_p of the soil and the pile

d) The slenderness ratio l_p/r_o in which l_p is pile length

e) Material damping of both soil and pile

f) The pile's tip restraint condition and rotational fixity of the head

g) Variation of soil and pile properties with depth

These factors affecting the functions f are not of equal importance in all situations. Often, some of them can be neglected, making it possible to present numerical values of the functions f in the form of tables and charts for some basic cases.

The pile stiffness diminishes with frequency quickly if the soil is very weak relative to the pile. This happens when the soil shear modulus is very low or when the pile is very stiff. In addition, dynamic stiffness can be considered practically independent of frequency for slender piles in average soil.

The imaginary part of the impedance (pile damping) grows almost linearly with frequency and, therefore, can be represented by constants of equivalent viscous damping c_i, which are also almost frequency-independent. Only below the fundamental natural frequencies of the soil layer (Eq. (5.4.2a) and (5.4.2b)) does geometric damping vanish and material damping remain as the principal source of energy dissipation. In particular, Eq. (5.4.2a) applies to foundation vibrations producing horizontal motion (for example, horizontal and torsion vibrations), and Eq. (5.4.2b) applies to foundation vibrations producing vertical motion (for example, vertical and rocking vibrations). Soil damping can then be evaluated using Eq. (5.4.2c) and (5.4.2d). The disappearance of geometric damping may be expected for low frequencies and shallow layers, stiff soils, or both. Apart from these situations, frequency-independent viscous damping constants, and functions f_{i2}, which define them, are sufficient for practical applications.

The mass density ratio ρ/ρ_p is another factor whose effect is limited to extreme cases. Only for very heavy piles do the pile stiffness and damping change significantly with the mass ratio.

The Poisson's ratio effect is very weak for vertical vibration, absent for torsion, and not very strong for the other modes of vibration, unless the Poisson's ratio approaches 0.5 and frequencies are high. The effect of Poisson's ratio on parameters f_{i1} and f_{i2} can be further reduced if the ratio E/E_p, rather than G/E_p, is used to define the stiffness ratio.

The slenderness ratio l_p/r_o and the tip conditions are very important for short piles, particularly for vertical motion because the piles are stiff in that direction. Floating piles (also called friction piles) have lower stiffness but higher damping than end bearing piles. In the horizontal direction, piles tend to be very flexible. Consequently, parameters f_{i1} and f_{i2} become practically independent of pile length and the tip condition for l_p/r_o greater than 25 if the soil medium is homogeneous.

Observations suggest that the most important factors controlling the stiffness and damping functions f_{i1} and f_{i2} are the stiffness ratio relating soil stiffness to pile stiffness, the soil profile, and, for the vertical direction, the tip restraint condition. It should be noted that the stiffness and damping functions f_{i1} and f_{i2} reported by Novak (1974) were obtained using plain strain models, which introduce substantial underestimation of stiffness and overestimation of damping values at frequencies $\omega d/V_s < 1$ (Gazetas et al. 1993). Additionally, the lateral radiation damping exhibits spurious high sensitivity to Poisson's ratio, which is mainly caused by the restriction of vertical soil deformation introduced by the plain strain model (Gazetas et al. 1993). Therefore, it is recommended that dynamic pile impedance be calculated using finite element models or computer codes specifically developed for this purpose. Alternatively, the approach proposed by Gazetas et al. (1991, 1993) and Mylonakis and Gazetas (1998, 1999) could be used if the scope of the foundation design does not warrant more sophisticated techniques.

5.5.2 *Pile groups*—Piles are usually used in a group. The behavior of a pile group depends on the distance between individual piles. When the distance between individual piles is large (20 diameters or more), the piles do not affect each other, and the group stiffness and damping are the sums of the contributions from the individual piles. If, however, the piles are closely spaced, they interact with each other. This pile-soil-pile interaction or group effect exerts a considerable influence on the stiffness and damping of the group.

5.5.2.1 *Pile interaction neglected*—When spacing between piles reaches 20 diameters or more, the interaction between piles can be neglected. Then, the stiffness and damping of the pile group can be determined by the summation of dynamic stiffness and damping of the individual piles. In many cases, initial calculations are performed neglecting the interaction. An overall group efficiency factor is then determined and applied to the summations.

In the vertical and horizontal directions, the summation is straightforward. For torsion and sliding coupled with rocking, the position of the center of gravity (CG) and the arrangement of the piles in plan are important. Thus, the group stiffness and damping with respect to rotation are a function of the horizontal, vertical, and moment resistance of individual piles and the pile layout.

Under the assumption of infinitely rigid pile cap behavior, if the pile group is rotated, an amount ψ about the axis passing through its CG (Fig. 5.5.2.1a), the head of pile j undergoes horizontal translation $u_j = \psi y_j$, vertical translation $v_j = \psi x_j$, and rotation $\psi_j = \psi$. For the torsional stiffness and damping of the group, the rotation η applied at the CG twists the pile by the same angle and translates its head horizontally by a distance equal to ηR (Fig. 5.5.2.1b). With these considerations and the notation shown in Fig. 5.5.1a and 5.5.1b, the stiffness and damping of the pile group, for individual motions, as referenced to the centroid of the pile group, are as follows

Vertical translation:

$$k_{gv} = \sum_{j=1}^{N} k_{vj} \tag{5.5.2.1a}$$

and

$$c_{gv} = \sum_{j=1}^{N} c_{vj} \tag{5.5.2.1b}$$

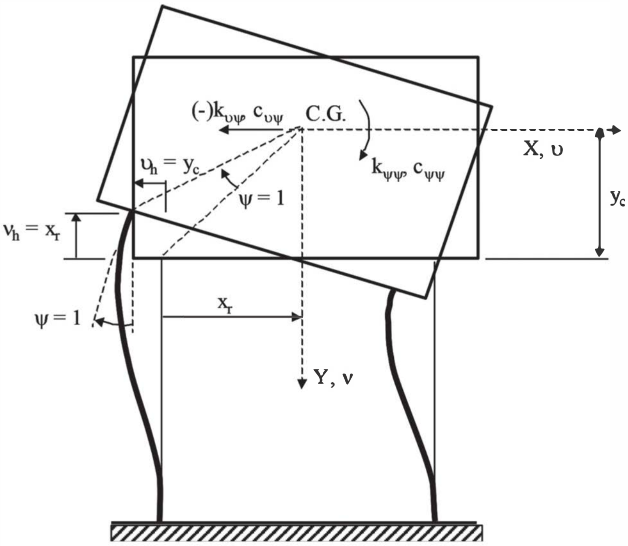


Fig. 5.5.2.1a—Pile displacements for determination of group stiffness and damping related to unit rotation ψ.

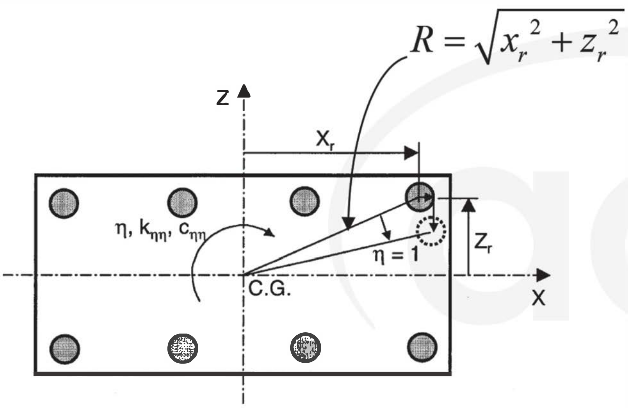


Fig. 5.5.2.1b—Pile displacements for determination of group stiffness and damping related to unit torsion η.

Horizontal translation:

$$k_{gu} = \sum_{j=1}^{N} k_{uj} \quad (5.5.2.1c)$$

and

$$c_{gu} = \sum_{j=1}^{N} c_{uj} \quad (5.5.2.1d)$$

Rocking about vertical plane:

$$k_{g\psi} = \sum_{j=1}^{N} (k_{\psi j} + k_{vj} x_j^2) \quad (5.5.2.1e)$$

and

$$c_{g\psi} = \sum_{j=1}^{N} (c_{\psi j} + c_{vj} x_j^2) \quad (5.5.2.1f)$$

Coupling between horizontal translation and rotation:

$$k_{gu\psi} = k_{g\psi u} = \sum_{j=1}^{N} k_{u\psi j} \quad (5.5.2.1g)$$

and

$$c_{gu\psi} = c_{g\psi u} = \sum_{j=1}^{N} c_{u\psi j} \quad (5.5.2.1h)$$

Torsion about vertical axis:

$$k_{g\eta} = \sum_{j=1}^{N} [k_{nj} + k_{uj}(x_j^2 + z_j^2)] \quad (5.5.2.1i)$$

and

$$c_{g\eta} = \sum_{j=1}^{N} [c_{nj} + c_{uj}(x_j^2 + z_j^2)] \quad (5.5.2.1j)$$

where the pile cap is assumed infinitely rigid. The summations extend over all the piles. The distances x_j and z_j refer to the distances from the centroid of the pile group to the individual pile. If the CG is located directly above the pile group centroid, these distances are as indicated in Fig. 5.5.2.1a and 5.5.2.1b. The vertical eccentricity y_c should be addressed as presented in 5.6. These stiffness and damping terms, or their impedance equivalents, represent values comparable to the terms developed for a soil-supported foundation in 5.4.

5.5.2.2 *Pile interaction considered*—When spacing between piles is less than 20 diameters, they interact with each other because the displacement of one pile contributes to the displacements of others. Studies of these effects call for the consideration of the soil as a continuum. The studies of the dynamic pile-soil-pile interaction provide the following findings:

a) Dynamic group effects are profound and differ considerably from static group effects.

b) Dynamic group effect is more pronounced as the number of piles increases.

c) Dynamic stiffness and damping of piles groups vary with frequency, and these variations are more dramatic than with single piles.

d) Group stiffness and damping can be either reduced or increased by pile-soil-pile interaction.

These effects can be demonstrated if the group stiffness and damping are described in terms of the group efficiency ratio (GE) defined as (Gazetas et al. 1993)

$$\text{group efficiency} = \frac{\text{dynamic group impedance}}{\text{static group stiffness}} \quad (5.5.2.2a)$$

Vertical translation:

$$K_v = \frac{k_{gv}}{\sum k_{vj}^{st}} \quad (5.5.2.2b)$$

and

$$D_v = \frac{c_{gv}}{\sum k_{vj}^{st}} \qquad (5.2.2.2c)$$

Horizontal translation:

$$K_u = \frac{k_{gv}}{\sum k_{uj}^{st}} \qquad (5.5.2.2d)$$

and

$$D_u = \frac{c_{gu}}{\sum k_{uj}^{st}} \qquad (5.5.2.2e)$$

Rocking about vertical:

$$K_\psi = \frac{k_{g\psi}}{\sum k_{\psi j}^{st} + k_{vj}^{st} x_j^2} \qquad (5.5.2.2f)$$

and

$$D_\psi = \frac{c_{g\psi}}{\sum k_{\psi j}^{st} + k_{vj}^{st} x_j^2} \qquad (5.5.2.2g)$$

where k_{ij}^{st} is the static stiffness of individual pile j in the i-th direction when considered in isolation. When dynamic pile-soil-pile interaction effects are absent, GE = 1. However, as illustrated in Fig. 5.5.2.2a to 5.5.2.2c (Gazetas et al. 1993), dynamic group effects are significant and cannot be ignored for the typical pile spacing used in practice ($s < 5d$). These figures also highlight the fact that static interaction factors (Poulos and Davis 1980) cannot be used for predicting the dynamic response of pile groups, except at very low frequencies of excitation (Gazetas et al. 1993). For instance, dynamic group efficiencies can be significantly larger than one and may become negative at certain frequencies whereas the static efficiency is always below unity, particularly for pile groups with large numbers of piles (Gazetas et al. 1993).

In summary, dynamic group effects are complex and there is no simple way of alleviating these complexities. Use suitable computer programs to describe the dynamic group stiffness and damping over a broad frequency range (Novak and Sheta 1982).

5.5.2.3 *Pile group stiffness using dynamic interaction coefficients*—An accurate analysis of dynamic behavior of pile groups should be performed using a suitable computer program. Nevertheless, a simplified approximate analysis suitable for hand calculations can be formulated based on dynamic interaction factors α. The interaction factors derive from the deformations of two equally loaded piles and give the fractional increase in deformation of one pile due to the deformation of an equally loaded adjacent pile. The flexibility and stiffness are then established by superposition

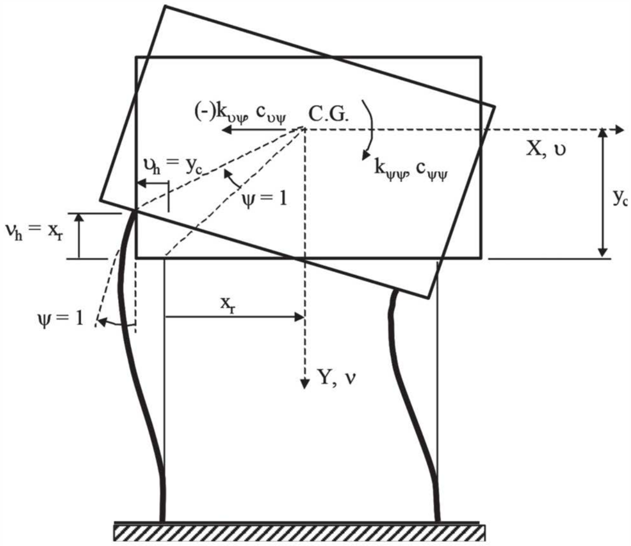


Fig. 5.5.2.2a—Normalized vertical dynamic stiffness and damping group factors of n x n *rigidly-capped pile groups in a nonhomogeneous halfspace. (E_p/E_s = 5000, L/d = 15, s/d = 5, ρ_s/ρ_p = 0 .7, β = 0.05 and ν = 0.4): effect of pile group configuration.*

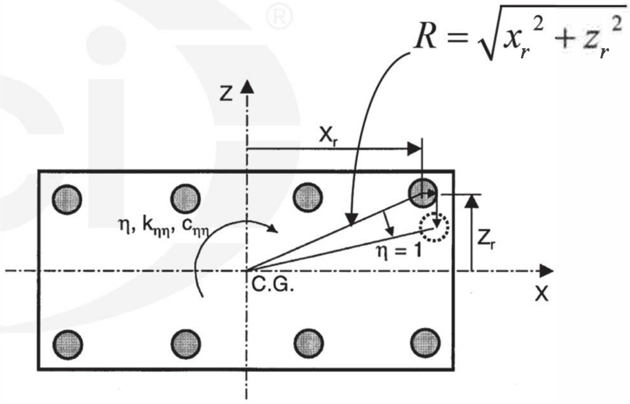


Fig. 5.5.2.2b—Normalized horizontal dynamic stiffness and damping group factors of n x n *rigidly-capped pile groups in a nonhomogeneous halfspace. (E_p/E_s= 5000, L/d = 15, s/d = 5, ρ_s/ρ_p = 0 .7, β = 0.05 and ν = 0.4): effect of pile group configuration.*

of the interaction between individual pairs of piles in the group. The approximation comes from neglecting the wave reflections of the other piles when evaluating the factor α. The accuracy of the approach appears adequate, at least for small to moderately large groups (Dobry and Gazetas 1988; Mylonakis and Gazetas 1998, 1999).

Dobry and Gazetas (1988) proposed a simple model for calculating the dynamic interaction factor between floating piles in homogeneous soil. According to that model, the response of the receiver pile to the oscillations of the source pile is equal approximately to the response of the free-field soil at the location of the receiver pile. Based on the subject assumption, the interaction factors are defined and calculated as follows

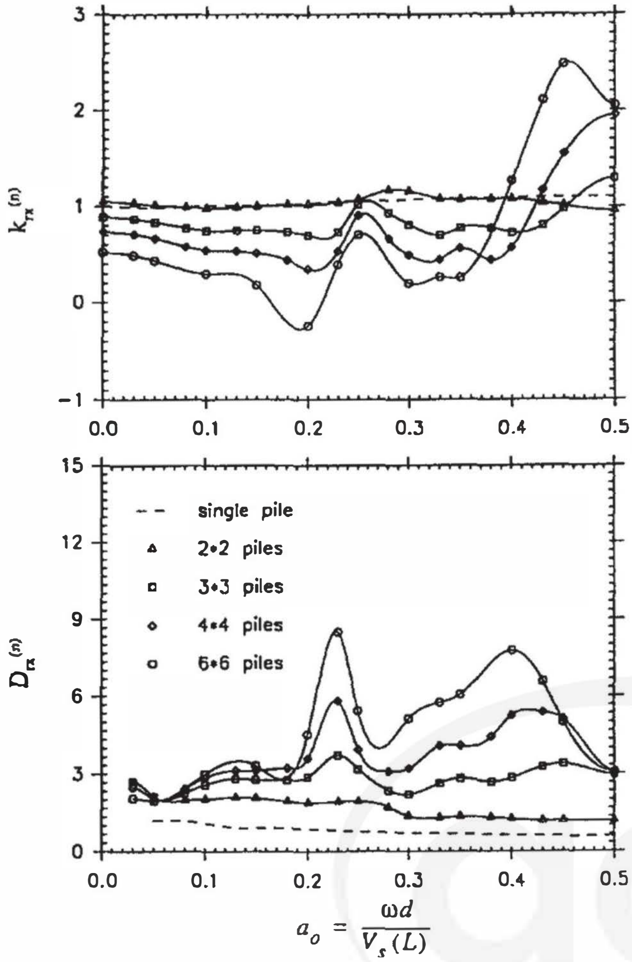


Fig. 5.5.2.2c—Normalized rotational dynamic stiffness and damping group factors of n x n *rigidly-capped pile groups in a nonhomogeneous halfspace.* (E_p/E_s = *5000,* L/d = *15,* s/d = *5,* ρ_s/ρ_p = *0.7,* β = *0.05 and* ν = *0.4): effect of pile group configuration.*

$$\alpha_i = \frac{\text{additional displacement of pile } q \text{ caused by pile } p}{\text{displacement of pile } q \text{ under own dynamic load}} \quad (5.5.2.3a)$$

Interaction factor α_z for axial in-phase oscillations of the two piles

$$\alpha_z \approx \sqrt{2}\left(\frac{s}{d}\right)^{-1/2} e^{-\beta_m \omega s/V_s} \cdot e^{-i\omega s/V_s} \quad (5.5.2.3b)$$

Interaction factor α_{HH} for lateral in phase oscillation

$$\alpha_{HH}(90°) = (3/4)\alpha_z$$

$$\alpha_{HH}(0°) = 0.5\left(\frac{s}{d}\right)^{-1/2} \cdot e^{-\beta_m \omega s/V_{La}} \cdot e^{-i\omega s/V_{La}}$$

$$\alpha_{HH}(\theta°) = \alpha_{HH}(0°)\cos^2\theta + \alpha_{HH}(90°)\sin^2\theta \quad (5.5.2.3c)$$

Interaction factors:

α_{MM} for in phase rocking, α_{MH} for swaying-rocking,

$$\alpha_{MM} \approx \alpha_{MH} \approx 0 \quad (5.5.2.3d)$$

where $V_{La} = 3.4V_s/\pi(1 - \nu)$ = Lysmer's analog velocity.

Although the interaction factors given by Eq. (5.5.2.3a) are complex numbers, their use is identical to the familiar static interaction factors (Poulos and Davis 1980). To calculate the dynamic stiffness of a pile group using the interaction factors approach, the impedance functions of single piles and the interaction factors are calculated first, then the group impedance functions are calculated for each machine frequency of interest. The stiffness and damping constants of individual piles are calculated per the procedures discussed in 5.5.1. The interaction factors are calculated using Eq. (5.5.2.3). The impedance functions of a pile group of n piles are then given by (El Naggar and Novak 1995)

Vertical group impedance:

$$K_v^G = k_v \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{ij}^v \quad (5.5.2.3e)$$

Horizontal group impedance:

$$K_h^G = k_u \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{2i-1,2j-1}^h \quad (5.5.2.3f)$$

Rocking group impedance:

$$K_\psi^G = k_h \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{2i,2j}^h \quad (5.5.2.3g)$$

Coupling group impedance:

$$K_c^G = k_h \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{2i-1,2j}^h \quad (5.5.2.3h)$$

where $[\varepsilon^v] = [\alpha]_v^{-1}$ (where α_{ij}^v is complex interaction factors between piles i and j for vertical translations) and $[\varepsilon^h] = [\alpha]_h^{-1}$ (where α_{ij}^h is complex interaction coefficients for the horizontal translations and rotations). The formulation of the $[\alpha]_h$ can be found in El Naggar and Novak (1995).

5.5.2.4 *Pile group stiffness using static interaction coefficients*—In place of the approach outlined in 5.5.2.3, pile static interaction coefficients may be used to estimate dynamic pile group stiffness if the frequency of interest is low. If the dimensionless frequency a_o < 0.1, or if the frequency is much less than the natural frequency of the soil layer as determined by Eq. (5.4.2a) and (5.4.2b), then this approach should provide a reasonable estimate of pile group stiffness.

5.5.3 *Battered piles*—Pile batter can be considered by calculating the pile stiffnesses for a vertical pile, assembling the stiffness as the stiffness matrix $[K]$ in element coordinates (along and perpendicular to the axis of the pile), and transforming this matrix into horizontal and vertical global coordinates (Novak 1979). This gives the pile stiffness matrix

$$[k']_i = [T]^T[k]_i[T] \quad (5.5.3a)$$

in which the transformation matrix $[T]$ depends on direction cosines from the batter. The horizontal and vertical stiffnesses for the individual battered pile can then be combined with the other pile stiffnesses, as presented in 5.5.2 for pile groups.

When the horizontal coordinate axis lies in the plane of the batter, the transformation matrix is

$$[T] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5.5.3b)$$

The pile stiffness matrix in global coordinates becomes

$$\begin{bmatrix} k'_u & k'_{uv} & k'_{u\psi} \\ k'_{vu} & k'_v & k'_{v\psi} \\ k'_{\psi u} & k'_{\psi v} & k'_\psi \end{bmatrix} = \begin{bmatrix} k_u\cos^2(\alpha)+k_v\sin^2(\alpha) & \cos(\alpha)\sin(\alpha)(k_u-k_v) & \cos(\alpha)k_{u\psi} \\ \cos(\alpha)\sin(\alpha)(k_u-k_v) & k_u\sin^2(\alpha)+k_v\cos^2(\alpha) & \sin(\alpha)k_{u\psi} \\ \cos(\alpha)k_{\psi u} & \sin(\alpha)k_{\psi u} & k_\psi \end{bmatrix} \quad (5.5.3c)$$

The element stiffness functions k are calculated assuming that the pile is vertical, thus

$$k_{uv} = k_{\psi v} = k_{v\psi} = 0 \text{ and } k_{u\psi} = k_{\psi u} \quad (5.5.3d)$$

In some cases, the off-diagonal terms of the transformed stiffness matrix may be ignored and only the diagonal terms carried forward. One criterion for this is if the off-diagonal terms are small in comparison to the diagonal terms (for example, less than 10 percent on an absolute value comparison). The same process may be applied to the damping terms, or, for more accuracy, the transformation can take place using the complex impedance functions. Dynamic group effects on battered piles are complex and should be calculated using specialized computer codes.

5.6—Transformed impedance relative to center of gravity

A machine foundation implies a body of certain depth, and, typically, the center of gravity of the system is above the center of resistance provided by the soil, embedment, piles, and any combination of these. For simplicity of analysis, many foundations are treated as a rigid body, and their center of gravity is used as the point of reference for all displacements and rotations. Thus, stiffness or impedance provided by the support system (piles, isolation springs, soil, or soil embedment) should be transformed to reflect resistance provided against motions of the center of gravity. In this analysis, a horizontal translation is resisted not only by horizontal soil reactions, but also by moments. This gives rise to a coupling between translation and rotation and the corresponding off-diagonal or cross stiffness and damping constants such as $k_{u\psi} = k_{\psi u}$ and $c_{u\psi} = c_{\psi u}$. Double subscripts are used to indicate coupling. Equations (5.5.1h) and (5.5.1k) also introduced coupling of terms.

For coupled horizontal and rocking motions, the generation of the stiffness and damping constants is developed as shown in Fig. 5.6. By applying unit translations to a free body of the foundation, examining the forces developed in the support system by the translation, and determining the forces needed to cause this unit translation, the coupled impedances can be determined with respect to the center of gravity. Figure 5.6 is presented for a simple system where both embedment and bottom support are provided. Evaluation of the forces associated with free-body movements yields the following impedance matrix for the system with respect to the center of gravity:

$$\begin{bmatrix} K^*_{uu} & K^*_{u\psi} \\ K^*_{u\psi} & K^*_{\psi\psi} \end{bmatrix} = \begin{bmatrix} k^*_u + k^*_{eu} & -(k^*_u y_c + k^*_{eu} y_e) \\ -(k^*_u y_c + k^*_{eu} y_e) & (k^*_\psi + k^*_u y_c^{\,2} + k^*_{eu} y_e^{\,2}) \end{bmatrix} \quad (5.6)$$

If the CG is not directly over the center of vertical impedance, additional coupling terms between the rotation and the vertical motion are introduced. Thus, most engineers diligently adhere to guidelines to minimize such in-plan eccentricities. In extreme cases, it may be appropriate to develop the K^* matrix as a full six-by-six matrix due to eccentricities. For other combinations of directions in other coordinate systems, the sign on the off-diagonal terms may change, so close attention to sign convention is required. This transformation to the CG may be developed on stiffness and damping terms or based on impedance.

5.7—Added mass concept

The dynamic stiffness of surfaces and pile foundations may become negative for certain frequencies, as shown in previous sections. This is because the dynamic stiffness captures the rigidity and inertia of the vibrating soil, as discussed in 5.2 and Eq. (5.2f). Typical structural codes do not have the capability to conduct analysis using the complex frequency response method and, therefore, cannot deal with negative springs directly. In this case, the concept of added mass can be used when dealing with negative springs. This is done by analogy with the single-degree-of-freedom (SDOF) system studied in 5.2. For this purpose, Eq. (5.3.2d) is rewritten and shown in Eq. (5.7). Additional details regarding the concept of added mass/inertia to model frequency-dependent impedance functions can be found in Saitoh (2007)

$$k(\omega) = \text{Re}(k^*(\omega)) = k_{st} - m^s\omega^2 = k_{si} - m_{si}\omega^2 \quad (5.7)$$

where $k(\omega) = k_i(\omega)$ is dynamic stiffness coefficient of the foundation for vibration mode i and machine frequency ω (that is, k_v, k_u, k_ϕ, k_η, etc.); $k_{si} = k_i(0) = k_i^*(0)$ is static stiffness coefficients of the foundation (that is, k_{sv}, k_{su}, $k_{s\phi}$, $k_{s\eta}$); and $m_{si} = [k_{si} - k_i(\omega)]/\omega^2$ is added inertia for vibration mode i at frequency ω_μ (that is, m_{sv}, m_{su}, $I_{s\phi}$, J_η).

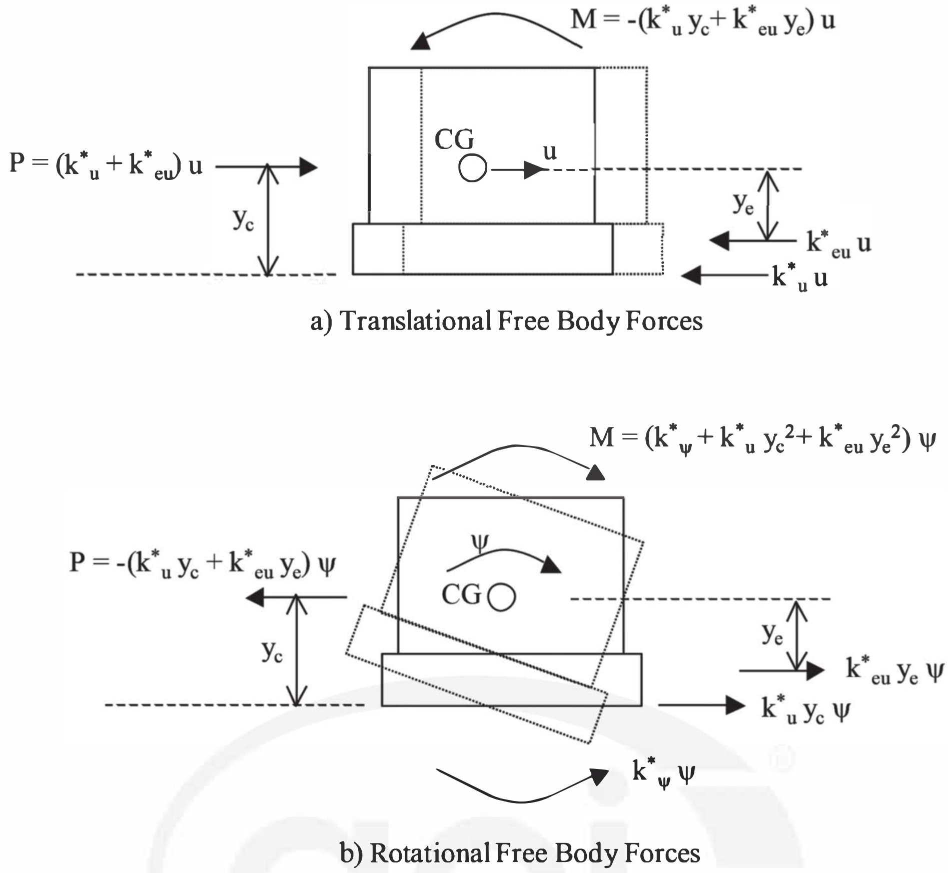


Fig. 5.6—Coupling effect introduced by an elevated CG.

5.8—Sample impedance calculations

5.8.1 *Shallow foundations*—This section presents numerical calculations using the equations from the previous sections in Chapter 5. Only the vertical impedance is included; the other directions follow the same approach. These calculations are not typical of all machine foundations and, as such, broad conclusions about results from alternate equations should not be reached. The results of these various calculations are given in Table 5.8.1a.

Given quantities:

Soil shear modulus G = 10 ksi = 1,440,000 lbf/ft^2 (69,000,000 N/m^2)

Soil Poisson's ratio ν = 0.45

Soil weight density w = 120 lbf/ft^3 (18.86 kN/m^3)

Material damping β_m = 5 percent, or 0.05

Soil is cohesive

Mat foundation width a = 20 ft (6m)

Mat foundation length b = 15 ft (4.5m)

Machine speed = 350 rpm

Effective embedment depth = 3 ft (1m)

Base calculations:

Circular operating frequency ω_o:

$$\omega_o = (350\text{ rpm})(2\pi\text{rad/rev})(1/60\text{ min/s}) = 36.65\text{ rad/s} \quad (5.8.1a)$$

Equivalent radius for vertical vibration:

$$R = \sqrt{\frac{ab}{\pi}} = \sqrt{\frac{(20\text{ ft})(15\text{ ft})}{\pi}} = 9.77\text{ ft (2.93 m)} \quad (5.8.1b)$$

Soil mass density ρ:

$$\rho = w/\text{gravity} = (120\text{ lbf/ft}^3)/(32.2\text{ ft/s}^2) = 3.73\text{ lbf-s}^2/\text{ft}^4\ (1922.5\text{ kg/m}^3) \quad (5.8.1c)$$

Because this machine generates a harmonic dynamic force with a frequency that matches the machine speed, the frequency of the motion will match the operating speed, ω equals ω_o.

Nondimensional frequency:

$$a_o = \omega_o R\sqrt{\rho/G} \quad (5.8.1d)$$

$$a_o = (9.77\text{ ft})\left(36.65\frac{\text{rad}}{\text{s}}\right)\sqrt{\frac{\left(3.73\text{ lbf}\frac{\text{s}^2}{\text{ft}^4}\right)}{\left(1{,}440{,}000\frac{\text{lbf}}{\text{ft}^2}\right)}} = 0.576$$

Table 5.8.1a—Summary of vertical impedance calculations

	Calculation basis	k_v, kip/in. (kN/mm)	c_v, kip-s/in. (kN-s/mm)*	Comment
No embedment	Eq. (5.4.1.2c)	8170 (1431)	110.1 (19.28)	No material damping
	Adjust by Eq. (5.4.4b) and (5.4.4c)	7770 (1630)	132.4 (23.2)	5 percent material damping
	Eq. (5.4.1.2g), Table 5.4.1.2	8790 (1439.3)	125.3 (21.94)	No material damping
	Adjust by Eq. (5.4.4b) and (5.4.4c)	8330 (1460)	149.3 (26.15)	5 percent material damping
	Eq. (5.4.1.1a), (5.4.1.1b), and (5.4.1.1i)	8530 (1495)	114 (20)	No material damping
	Adjust by adding 5% for material damping	8530 (1495)	120.4 (21.1)	5 percent material damping ∑Wt = 187.5 k
	Veletsos and Verbic (1973) complete	8030 (1406)	133.5 (23.4)	5 percent material damping included; best calculation
Embedment effects (additive to above)	Eq. (5.4.3b)	972 (170.25)	37.9 (6.65)	No material damping
	Adjust by Eq. (5.4.4b) and (5.4.4c)	833 (146)	40.6 (7.1)	5 percent side material damping

*Note: c_v values have not been reduced as discussed in 5.4.

Method 1: Base vertical impedance—Section 5.4.1.2, Eq. (5.4.1.2c)

From Table 5.8.1b:
(Values for 0.45 are interpolated between 0.33 and 0.5.)

v = 0.33	v = 0.5	v = 0.45
$\gamma_1 = 0.350$	0.000	0.103
$\gamma_2 = 0.800$	0.000	0.235
$\gamma_3 = 0.000$	0.170	0.120
$\gamma_4 = 0.750$	0.850	0.821

Parameter χ_v

$$\chi_v = \frac{\gamma_1(\gamma_2 a_0)^2}{1+(\gamma_2 a_0)^2} = \frac{(0.103)[(0.235)(0.576)]^2}{1+[(0.235)(0.576)]^2} = 0.00185 \quad (5.8.1e)$$

Parameter ψ_v

$$\psi_v = \frac{\gamma_1\gamma_2(\gamma_2 a_o)^2}{1+(\gamma_2 a_o)^2}$$

$$= \frac{(0.103)(0.235)[(0.235)(0.576)]^2}{1+[(0.235)(0.576)]^2} = 0.000436 \quad (5.8.1f)$$

Vertical impedance

$$k_v^* = \frac{4GR}{(1-v)}[(1-\chi_v-\gamma_3 a_o^2)+ia_o(\gamma_4+\psi_v)] \quad (5.8.1g)$$

$$k_v^* = \frac{4(1,440,000 \text{ lbf/ft}^2)(9.77 \text{ ft})}{(1-0.45)}$$
$$[(1-0.00185-(0.12)(0.576)^2)+i(0.576)(0.821+0.000436)]$$

Table 5.8.1b—Values of α_1, β_j, and γ_j

	v = 0	v = 0.33	v = 0.45	v = 0.50
α_1	0.775	0.650	0.600	0.600
β_1	0.525	0.500	0.450	0.400
β_2	0.800	0.800	0.800	0.800
β_3	0.000	0.000	0.023	.027
γ_1	0.850	0.750	—	0.850
γ_2	1.000	0.800	—	0.000
γ_3	0.000	0.000	—	0.170
γ_4	0.850	0.750	—	0.850

$$k_v^2 = 102,318,545[0.9583 + i0.4731]$$
$$\approx 98,100,000 + i48,400,000 \text{ lbf/ft}$$

The stiffness is equivalent to the real part of k_v^*

$$k_v = 98,100,000 \text{ lbf/ft or } 8170 \text{ kip/in. } (1431 \text{ kN/mm}) \quad (5.8.1h)$$

The damping constant is derived from the imaginary part of k_v^*

$$c_v = (48,400,000 \text{ lbf/ft})/(36.65 \text{ rad/s}) \approx 1,321,000 \text{ lbf-s/ft or } 110.1 \text{ kip-s/in. } (19.28 \text{ kN-s/mm}) \quad (5.8.1i)$$

Including material damping through Eq. (5.4.4b) and (5.4.4c) yields adjusted stiffness and damping terms of

$$k_v(\text{adj}) = k_v - 2\beta_m \times c_v \times \omega_o \quad (5.8.1j)$$

$$k_v(\text{adj}) = 98,100,000 - 2(0.05)(1,321,000)(36.65)$$
$$\approx 93,300,000 \text{ lbf/ft} = 7770 \text{ kip/in. } (1630 \text{ kN/mm})$$

$$c_v(\text{adj}) = c_v + 2\beta_m \times k_v/\omega_o \quad (5.8.1k)$$

c_v(adj) = 1,321,000 + 2(0.05)(98,100,000)/(36.65)
≈ 1,589,000 lbf-s/ft = 132.4 kip-s/in. (23.2 kN-s/mm)

Method 2: Base vertical impedance—Section 5.4.1.2 and Eq. (5.4.1.2g), Table 5.4.1.2

Use the same input data as before. Use of Table 5.4.1.2 is allowed because a_o is less than 2.0. From Table 5.4.1.2, using cohesive soil, $C_{v1} = 7.5$, $C_{v2} = 6.8$

Vertical impedance

$$k_v^* = GR(C_{v1} + ia_oC_{v2}) = (1{,}440{,}000 \text{ lbf/ft}^2)(9.77 \text{ ft})[7.5 + i(0.576)(6.8)]$$

$$k_v^* \approx 105{,}500{,}000 + i55{,}100{,}000 \text{ lbf/ft} \quad (5.8.11)$$

The stiffness is equivalent to the real part of k_v^*

$$k_v = 105{,}500{,}000 \text{ lbf/ft or } 8790 \text{ kip/in. } (1539.3 \text{ kN/mm}) \quad (5.8.1m)$$

The damping constant is derived from the imaginary part of k_v^*

$$c_v = 55{,}100{,}000/36.65 \approx 1{,}503{,}000 \text{ lbf-s/ft or } 125.3 \text{ kip-s/in. } (21.9 \text{ kN-s/mm}) \quad (5.8.1n)$$

Calculations using polynomial expansions of C_{v1} and C_{v2} support the tabulated constants for this speed of operation (a_o = 0.576). For higher speeds, the difference between the constant value and the polynomial are significant.

Including material damping through Eq. (5.4.4b) and (5.4.4c) yields adjusted stiffness and damping terms of:

$$k_v(\text{adj}) = 8790 - 2(0.05)(125.3)(36.65) = 8330 \text{ kip/in. } (\approx 1460 \text{ kN/mm}) \quad (5.8.1o)$$

$$c_v(\text{adj}) = 125.3 + 2(0.05)(8790)/(36.65) = 149.3 \text{ kip-s/in. } (26.1 \text{ kN-s/mm}) \quad (5.8.1p)$$

Method 3: Base vertical impedance—Richart and Whitman (1967): Eq. (5.4.1.1a), (5.4.1.1b), and (5.4.1.1i)

This approach is valid for a_o less than 1.0. Assume the machine weighs 30 kip (135 kN) and the mat foundation is 3.5 ft (1100 mm) thick. These assumptions are not significant for the impedance in the vertical direction; the k_v and c_v values are not dependent on these weights. For rotational motions, there is some minor dependence on the specific assumed values.

The base stiffness is calculated as

$$k_v = 4GR/(1 - \nu) = 4(1{,}440{,}000 \text{ lbf/ft}^2)(9.77 \text{ ft})/(1 - 0.45) \approx 102{,}300{,}000 \text{ lbf/ft or } 8530 \text{ kip/in. } (\approx 1495 \text{ kN/mm}) \quad (5.8.1q)$$

The total weight of machine and foundation is

$$W = (20 \text{ ft})(15 \text{ ft})(3.5 \text{ ft})(0.15 \text{ kcf}) + (30 \text{ kip}) = 187.5 \text{ kip } (\approx 835 \text{ kN}) \quad (5.8.1r)$$

Either weights (W, w) or masses (m, ρ) can be used in Eq. (5.4.1.1a), provided consistency is maintained. The mass ratio for this calculation is

$$B_v = (1 - \nu)W/[4wR_o^3] = (1 - 0.45)(187.5 \text{ kip})/[4(0.12 \text{ kcf})(9.77 \text{ ft})^3] = 0.230 \quad (5.8.1s)$$

Damping ratio (geometric damping)

$$D_v = \frac{0.425}{\sqrt{B_v}} = \frac{0.425}{\sqrt{0.23}} = 0.886 = 88.6\% \quad (5.8.1t)$$

The total system mass is

$$M = \frac{W}{g} = \frac{187.5 \text{ kip}}{386.4 \text{ in./s}^2} = 0.485 \text{ kip-s}^2/\text{in. } (0.085 \text{ kN-s}^2/\text{mm}) \quad (5.8.1u)$$

The damping constant is calculated as

$$c_v = 2D_v\sqrt{(k_vM)} = 2(0.886)\sqrt{(8530 \text{ kip/in.})(0.485 \text{ kip-s}^2/\text{in.})} = 114 \text{ kip-s/in. } (20 \text{ kN-s/mm}) \quad (5.8.1v)$$

To include the material damping as a simple addition, the 5 percent material damping is added to the D_v value, and Eq. (5.4.1.1h) is applied. The stiffness is not altered in this approach.

$$c_v = 2D_v\sqrt{(k_vM)} = 2(0.936)\sqrt{(8530 \text{ kip/in.})(0.485 \text{ kip-s}^2/\text{in.})} = 120.4 \text{ kip-s/in. } (21.1 \text{ kN-s/mm}) \quad (5.8.1w)$$

Method 4: Base vertical impedance—Veletsos and Verbic (1973)

Veletsos and Verbic (1973) presents more complete versions of 5.4.1.2, Eq. (5.4.1.2c), that include material damping. Short of a complete complex domain solution, this approach is accepted as the best calculation basis. With material damping of 5 percent, those equations yield

k_v = 8030 kip/in. (1406 kN/mm)
c_v = 133.5 kip-s/in. (23.4 kN-s/mm)

Embedment vertical impedance—Eq. (5.4.3b)

Because the dimensionless frequency (a_o = 0.576) is in the range of 0.5 to 1.5, the use of Table 5.4.1.2 factors is permissible. From Table 5.4.1.2, $S_{v1} = 2.7$, $S_{v2} = 6.7$.

Embedment vertical impedance

$$k_{ev}^* = G_s l[S_{v1} + ia_o S_{v2}]$$
$$= (1{,}440{,}000 \text{ lbf/ft}^2)(3 \text{ ft})[(2.7 + i\,(0.576)(6.7)]$$
$$\approx 11{,}660{,}000 + i16{,}670{,}000 \text{ lbf/ft} \quad (5.8.1x)$$

The stiffness is equivalent to the real part of k_{ev}^*

$$k_v = 11{,}660{,}000 \text{ lbf/ft or } 972 \text{ kip/in. } (170.25 \text{ kN/mm}) \quad (5.8.1y)$$

The damping constant is derived from the imaginary part of k_{ev}^*

$$c_v = (16{,}670{,}000 \text{ lbf/ft})/(36.65 \text{ rad/s})$$
$$= 455{,}000 \text{ lbf-s/ft or } 37.9 \text{ kip-s/in. } (6.65 \text{ kN-s/mm}) \quad (5.8.1z)$$

These values can be adjusted for the material damping effects of the embedment material. The material damping of this side material may be different from the base material. Using Eq. (5.4.4b) and (5.4.4c) yields adjusted stiffness and damping terms of

$$k_v(\text{adj}) = (972 \text{ kip/in.}) - 2\,(0.05)(37.9 \text{ kip-s/in.})(36.65 \text{ rad/s})$$
$$= (972 \text{ kip/in.}) - (139.0 \text{ kip/in.}) = 833 \text{ kip/in. } (146 \text{ kN/mm}) \quad (5.8.1aa)$$

$$c_v(\text{adj}) = (37.9 \text{ kip-s/in.}) + 2\,(0.05)(972 \text{ kip/in.})/(36.65 \text{ rad/s})$$
$$= (37.9 \text{ kip-s/in.}) + (2.7 \text{ kip-s/in.}) = 40.6 \text{ kip-s/in. } (7.1 \text{ kN-s/mm}) \quad (5.8.1bb)$$

The embedment impedance values are directly additive to the base impedance values. For example, combining with the results from the complete solution of Veletsos (Method 4 previously mentioned) yields

$$(k_v)_{total} = (k_v)_{base} + (k_v)_{embed} = (8030 \text{ kip/in.}) + (833 \text{ kip/in.})$$
$$\approx 8860 \text{ kip/in. } (1552 \text{ kN/mm}) \quad (5.8.1cc)$$

$$(c_v)_{total} = (c_v)_{base} + (c_v)_{embed} = (133.5 \text{ kip-s/in.}) + (40.6 \text{ kip-s/in.})$$
$$= 174.1 \text{ kip-s/in. } (30.5 \text{ kN-s/mm}) \quad (5.8.1dd)$$

Base impedance values from other calculations could also be used. Most often, consistent approaches are used. The overall results of these various calculations are tabulated in Table 5.8.1a.

Calculation of displacements

Once the stiffness and damping have been determined, the engineer calculates the maximum response using the single-degree-of-freedom (SDOF) approach in Chapter 6. For demonstration, use the Veletsos and Verbic (1973) values with the embedment terms added and reduce the damping by 50 percent as a rough consideration of possible soil layering effects. Thus, the embedded stiffness is 8860 kip/in. (1552 kN/m) and the embedded damping is 87 kip-s/in. (15.2 kN-s/mm). The total system weight is 187.5 kip (≈835 kN) (refer to Method 3 calculation).

To calculate motions, determining an excitation force is necessary. Use a given rotor weight of 10 kip and Eq. (5.8.1ee) to get

$$F = 10{,}000 \text{ lbf} \times 350/6000 = 583 \text{ lbf} \quad (5.8.1ee)$$

Calculating the natural frequency ω_n and damping ratio D,

$$\omega_n = (k/m)^{1/2} = (kg/W)^{1/2} \quad (5.8.1ff)$$

$$\omega_n = (8860 \text{ kip/in.} \times 32.2 \text{ ft/s}^2 \times 12 \text{ in./ft}/187.5 \text{ kip})^{1/2}$$
$$= 135.1 \text{ rad/s} \quad (5.8.1gg)$$

$$D = \frac{c}{2\sqrt{km}} = \frac{c}{2\sqrt{kW/g}} \quad (5.8.1hh)$$

$$D = \frac{87 \text{ kip-s/in.}}{2\sqrt{8860 \text{ kip/in.}\ \dfrac{187.5 \text{ kip}}{(32.2 \text{ ft/s}^2)(12 \text{ in./ft})}}} = 0.663 \quad (5.8.1ii)$$

The closed-form solution for a harmonically excited SDOF system is given by

$$A = \frac{F_o/k}{\sqrt{(1-(\omega_o/\omega_n)^2)^2 + (2D\omega_o/\omega_o)^2}} \quad (5.8.1jj)$$

$$A = \frac{583/8860}{\sqrt{(1-(36.65/135.1)^2)^2 + (2(0.663)(36.65)/135.1))^2}}$$
$$= \frac{0.0658}{0.994} = 0.0664 \text{ mils}$$

The predicted amplitude motion of this SDOF system is 0.0662 mils. The peak-to-peak motion is 0.1324 mils (3.36 mm) at 350 rpm. For comparison using Fig. 5.8.1a, this motion at this speed is seen to qualify as extremely smooth. Similarly, on Fig. 5.8.1b(a) and 5.8.1b(b), the resultant motion at the operating speed is well below the various corporate and other standards for acceptance. This figure provides a comparison of five company standards, designated A through E, plotted against the Baxter and Bernhard (1967) background. Permissible amplitudes are presented for reciprocating (recip) and for rotating (rot) machines.

Although a ω_n value is calculated for use in Eq. (5.8.1ff), this is not actually the system natural frequency because the value has been calculated using a stiffness value specific to motion at a specific frequency (350 rpm or 36.65 rad/s). With frequency-dependent impedance, as the frequency of excitation increases, the stiffness decreases. Determining the frequency at which maximum response occurs requires an iterative solution across a range of frequencies. For this problem, such an iteration shows that the maximum undamped response (comparable with the aforementioned ω_n value) occurs at a speed of 1030 rpm (108 rad/s) and the more realistic, maximum damped response occurs at 1390 rpm (146 rad/s).

5.8.2 *Pile foundations*—Sample calculations for pile foundations are not included herein for reasons of brevity.

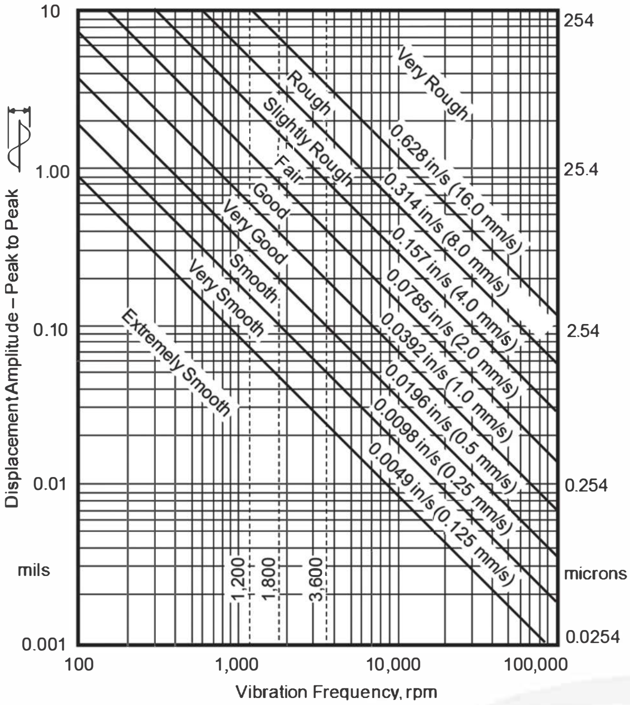


Fig. 5.8.1a—General machinery vibration severity chart (Baxter and Bernhard 1967).

Calculations for single piles can be found in a variety of sources including Novak (1974, 1977) and Kuhlmeyer (1979a,b). For pile groups, Dobry and Gazetas (1988), El Naggar and Novak (1995), Mylonakis and Gazetas (1998, 1999), and Gazetas et al. (1991, 1993) may be consulted for sample calculations.

CHAPTER 6—VIBRATION ANALYSIS AND ACCEPTANCE CRITERIA

6.1—Overview

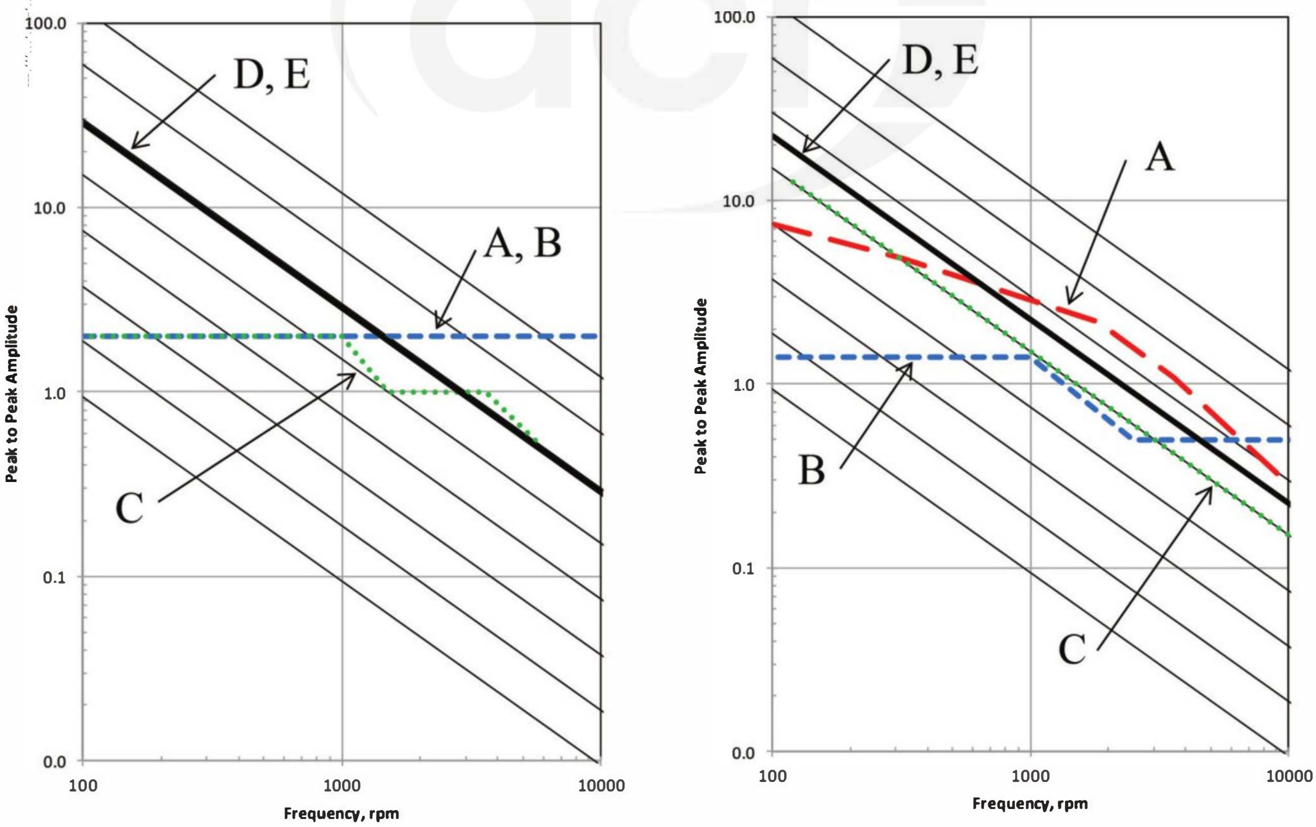
The purpose of this chapter is to describe analytical methods to determine the adequacy of foundations subject to dynamic machine forces. This is part of the overall design procedure described in Chapter 7.

A complete dynamic analysis of a machine foundation system is normally performed in two stages:

1. **Frequency analysis**: Calculation of the natural frequencies of a machine-foundation system and a comparison with the frequency of the applied harmonic force. A frequency analysis is covered in 6.5.

2. **Forced response analysis**: Calculation of the machine-foundation system amplitudes caused by the applied dynamic forces. A forced response analysis is covered in 6.6.

These two stages involve a solution of the equation of motion:



(a) Reciprocating (b) Rotating

Fig. 5.8.1b—Comparison of permissible displacements (mils) for five company standards.

$$M\ddot{y} + C\dot{y} + Ky = F(t) \quad (6.1)$$

The first term, $M\ddot{y}$, is the internal force due to inertia, consisting of the mass M and acceleration (or the second derivative of the displacement) $\ddot{y}$. Mass is calculated from the properties of the foundation and machine.

The second term, $C\dot{y}$, is the internal force due to viscous damping, consisting of the damping coefficient C and velocity (or first derivative of displacement) $\dot{y}$. Damping is determined from concrete properties, soil properties, and foundation configuration (refer to Chapter 5).

The third term, Ky, is the internal force due to stiffness, consisting of the stiffness K and displacement y. Stiffness is determined from soil properties and foundation configuration (refer to Chapter 5).

The term on the right, $F(t)$, is the time-varying external machine force. The force is a property of the operating machine (refer to Chapter 4).

Equation (6.1) is expressed in a single-degree-of-freedom (SDOF) format. Multiple (coupled) degrees of freedom result from the interaction of translational and rotational movements, and the rigidity of the supporting concrete elements. The configuration of the foundation under analysis will determine the number of coupled degrees of freedom. This topic is examined further in the following section.

6.2—Modeling for rigid foundations

In many applications, concrete provides the required inertial mass so that foundations are often massive and considered rigid (Fig. 6.2a). This, in conjunction with the rigid nature of many machines, can greatly simplify dynamic analysis of the machine-foundation system.

As described in 3.3.1, a rigid foundation is a situation where the machine and foundation are considered rigid with respect to the soil or piles. Mat foundation thickness criteria primarily serve to support this simplifying assumption. Clearly, this is a more complex problem than is addressed by simple rules-of-thumb. On soft soils, a thinner section may be sufficient, whereas on stiffer soils, a thicker section might be required to support the rigid body assumption. Gazetas (1983) provides some clarification in this regard.

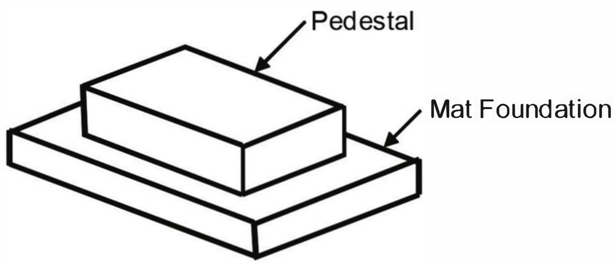
One rule-of-thumb criterion for thickness is that the minimum mat foundation thickness should be one-fifth of its width (short side), one-tenth of its length (long side), or 2 ft (0.6 m), whichever is greatest.

Another criterion for minimum thickness is

$$T_M > 2 + L_M/30 \text{ (ft)}$$
$$T_M > 0.6 + L_M/30 \text{ (m)} \quad (6.2)$$

If the mat foundation thickness is less than what is defined by the minimum, then the foundation is considered flexible in the vibration analysis and more elaborate calculation techniques, as described in 6.3, are justified.

Equation (6.2) is based on engineering judgment and industry practice for some traditional block-type foundations (Fig. 6.2a(a)). The applicability of Eq. (6.2) to large



(a) diagram

(b) actual foundation

Fig. 6.2a—Rigid foundation.

combined block-type foundations (Fig. 6.2b) is not well established and may need to be investigated by the engineer on a case-by-case basis. A finite element analysis may show that some of these large combined block-type foundations may be significantly flexible. Analysis of the frequency and dynamic response of such foundations using the finite element methods given in 6.3 will be more appropriate.

In general, a six DOF model is needed to properly represent the dynamic performance of a rigid machine-foundation system. In this situation, a rigid foundation will contain three translation modes (along the vertical and two lateral directions), and three rotation modes (about the vertical and two lateral directions). Solving six coupled equations of motion is typically handled by computer software.

If the foundation is well laid out so that the center of gravity (CG) and the center of resistance (CR) are vertically aligned, then the six DOF model mathematically uncouples to become two problems of two DOF (rocking about one horizontal axis coupled with translation along the other horizontal axis for both horizontal directions) and two SDOF problems (vertical motion and torsional motion about the vertical axis). The horizontal motion is coupled with the rocking motion due to the CG being at one height and the CR being at a different height. The advantage of this vertical alignment is that one and two DOF models can be solved manually (refer to Fig. 6.2c).

Sometimes a DOF is ignored if there are no dynamic forces acting in the DOF direction or if the ignored DOF is

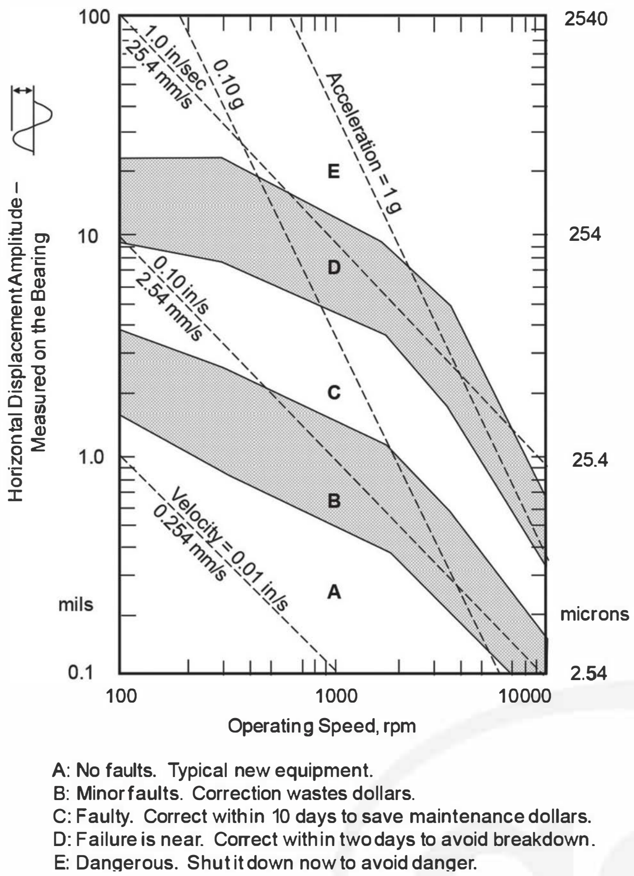


Fig. 6.2b—Vibration criteria for rotating machinery (Blake 1964, as modified by Arya et al. 1979).

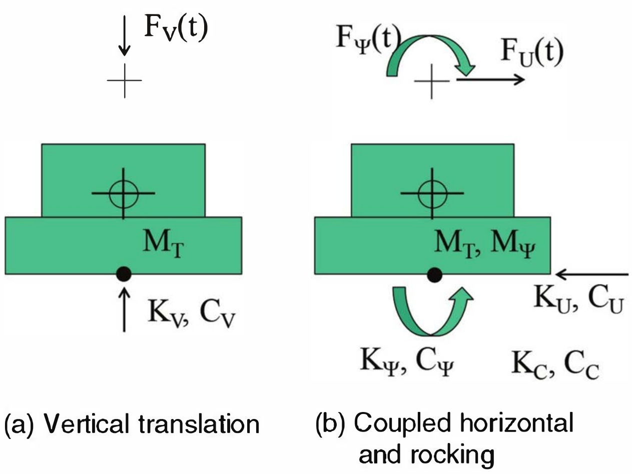


Fig. 6.2c—Aligned foundation representation. Note: Subscript U is horizontal, Ψ is rocking, and C is cross horizontal rocking.

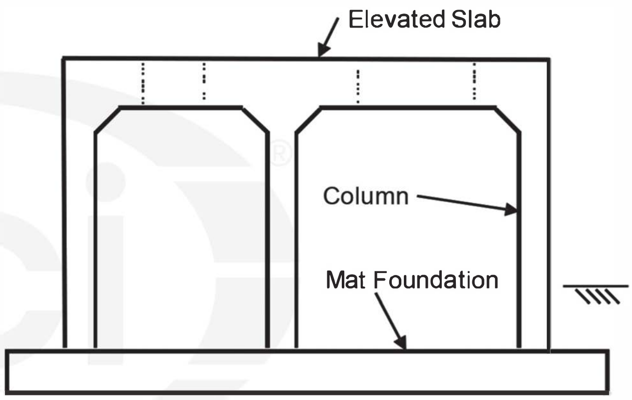


Fig. 6.3a—Flexible foundation.

relatively uncoupled with the dynamic forces. Examples of ignored DOF would be translation in the direction parallel to a rotating machine shaft, or rotation about the vertical axis.

6.3—Modeling for flexible foundations

A flexible foundation is any structural configuration that does not meet rigid foundation requirements. The classic case of a flexible foundation is a tabletop (Fig. 6.3a) that uses moment-resisting frames to provide open space for piping and equipment located directly below the machine. In this situation, a multi degree of freedom (MDOF) model should be used.

Equation (6.1), when expressed in MDOF format, becomes

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{F(t)\} \quad (6.3)$$

where [] represents a square matrix and {} represents a column matrix. The size of the matrixes is equal to the number of unknown degrees of freedom.

Structural members (beams, columns, and elevated slabs) are generally represented in the computer model by prismatic elastic members/elements, and concrete slabs and mat foundations are represented by plates or solid elements. The choice of modeling complexity can vary greatly. Practitioners use anything from simple beam elements to three-dimensional solid elements (Fig. 6.3b).

Structural stiffness comes from the modeled structural elements, in addition to specified lumped stiffness values applied to mat foundation and subgrade (soil) interface nodes. Structural damping comes from a specified damping ratio applied to the modeled structural elements, in addition to specified lumped damping coefficients applied to mat foundation and subgrade (soil) interface nodes.

The machine can be quite stiff compared with the structural members. If so, the machine can be represented by a mass point or points concentrated at their centers of gravity and connected to their supports using rigid links. Solid elements with matching densities can also be used to simulate the mass and mass moment of inertia of the machine. If the equipment is not so stiff as compared with the structural members, as is the case for many large gas compressors, its flexibility should be included in the computer model representation. Mat foundations and pile caps are commonly proportioned to be rigid to match assumptions made in the determination of soil or pile impedance.

Impedance of the soil or piles should be properly included in the model to account for the soil-structure interaction effects. Chapter 5 discusses the impedance of the supporting media. This impedance is nearly always included, either by

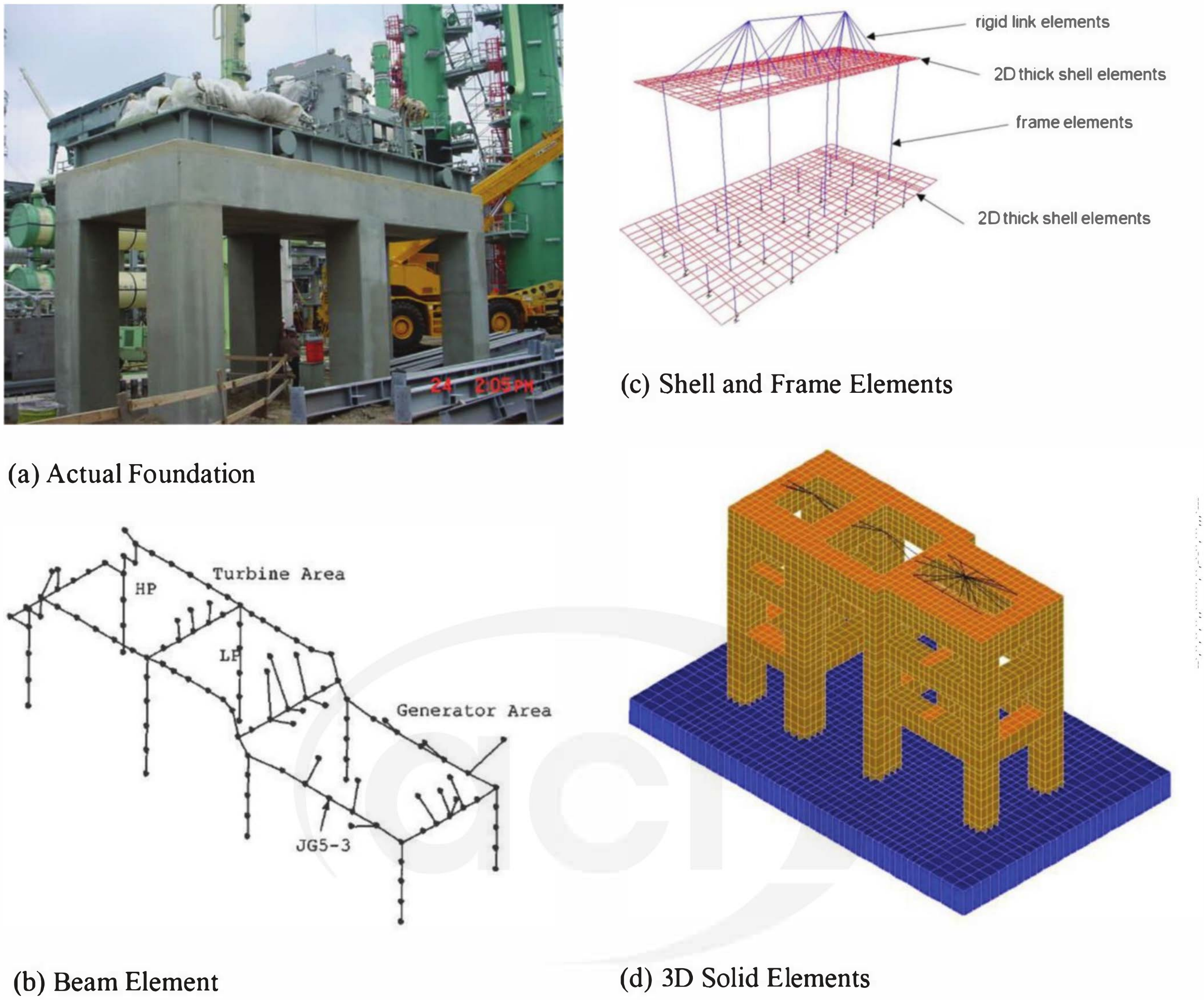


Fig. 6.3b—Foundation modeling complexity.

directly modeling the soil with the structure (6.3.1) or by separating concrete structure and soil/pile models (6.3.2).

The dynamic concrete modulus of elasticity is higher (that is, stiffer) than the static modulus, although not in any simple form. The distinction is more important for elevated tabletop-type foundations where the frame action of the structure is stiffer if a higher dynamic modulus is used. The difference can also be important for large compressors where the stiffness of the machine frame should be evaluated against the stiffness of the concrete structure. For simple, in-block-type foundations, the concrete modulus of elasticity has no real effect on the design. Refer to Eq. (7.2.3.1) for values.

6.3.1 *Combined model*—Soil/pile and machine/foundation elements may be combined into a single finite element model (Fig. 6.3.1). The advantage is that the interaction between soil and structure is handled automatically through the model solution. A second advantage is in the flexibility of model geometry. The disadvantage is the increased modeling complexity and additional solution time. In addition, because of the lack of machine symmetry, the axisymmetric soil models described in Chapter 5 would not be applicable.

6.3.2 *Substructured model*—Substructured models are more commonly used, where machine/foundation and soil/pile properties are evaluated separately (Fig. 6.3.2a). The soil evaluation can use a finite element model, or use one of the other procedures described in Chapter 5. The stiffness and damping from the soil or pile impedance evaluation would then be incorporated in the structure finite element model.

Soil/pile impedance is normally applied to the machine/foundation model as point values on interface mat foundation and subgrade node points. Pile impedance is normally applied as point values at nodes representing the individual pile locations. Soil impedance is normally applied in a similar manner, at nodes located to represent equal areas of soil pressure across the mat foundation area. Because the impedance of the supporting media is typically evaluated as a lumped value, this lumped value should then be divided for use at the selected node locations (refer to Fig. 6.3.2b).

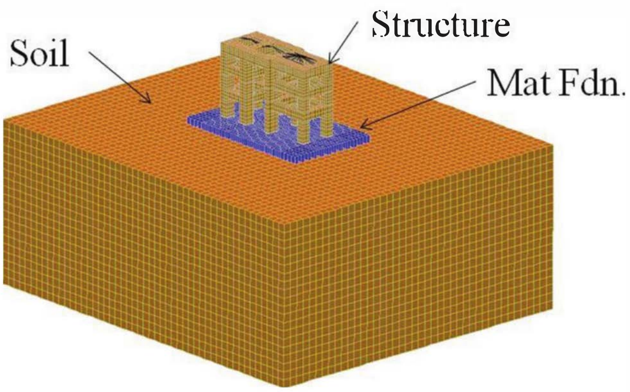


Fig. 6.3.1—Combined model.

An approximate method for this distribution would be to assume the mat foundation to be rigid and divide the lumped soil (or pile group) impedance value evenly such that the combined effect of the individual nodes is equivalent. The assumption of mat foundation rigidity would need to be confirmed; however, this assumption is typically inherent in the determination of soil impedance. This approach results in the following equations.

For stiffness:

$$k_v = K_v/N \quad (6.3.2a)$$

$$k_u = K_u/N \quad (6.3.2b)$$

$$k_\psi = [K_\psi - k_v(\sum L_P^2)]/N \quad (6.3.2c)$$

$$k_c = K_c/N \quad (6.3.2d)$$

For damping:

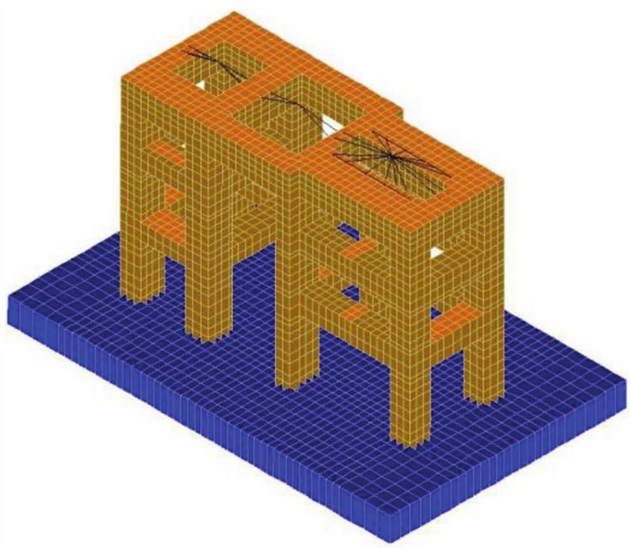
$$c_v = C_v/N \quad (6.3.2e)$$

$$c_u = C_u/N \quad (6.3.2f)$$

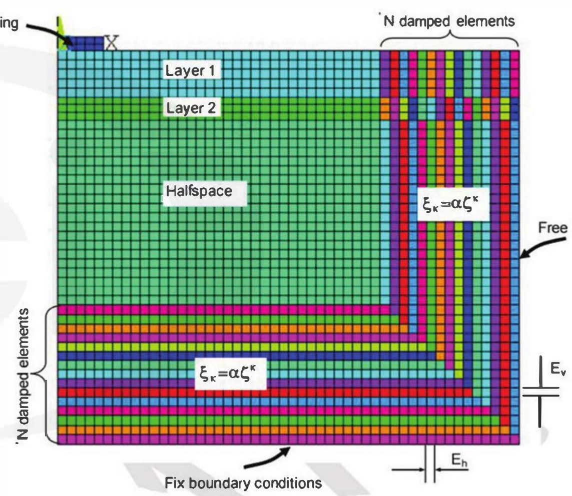
$$c_\psi = [C_\psi - c_v(\sum L_P^2)]/N \quad (6.3.2g)$$

$$c_c = C_c/N \quad (6.3.2h)$$

Computer programs used to calculate foundation impedance often report impedance at the center of gravity of the foundation plus machine (instead of the center of foundation resistance) unless the data input dictates otherwise. The user needs to be aware of this difference. In the aforementioned equations, subscript v represents vertical, subscript u represents horizontal, subscript ψ represents rocking, and subscript c represents cross horizontal-rocking values. The sign of cross impedance values depends on the axis orientation and can be different between the software used to calculate the impedance and the finite element software. Because the input of stiffness and damping into a finite element model can be different from the positive global axes, values and orientation should be carefully verified.



(a) Machine/Foundation Model



(b) Soil Model

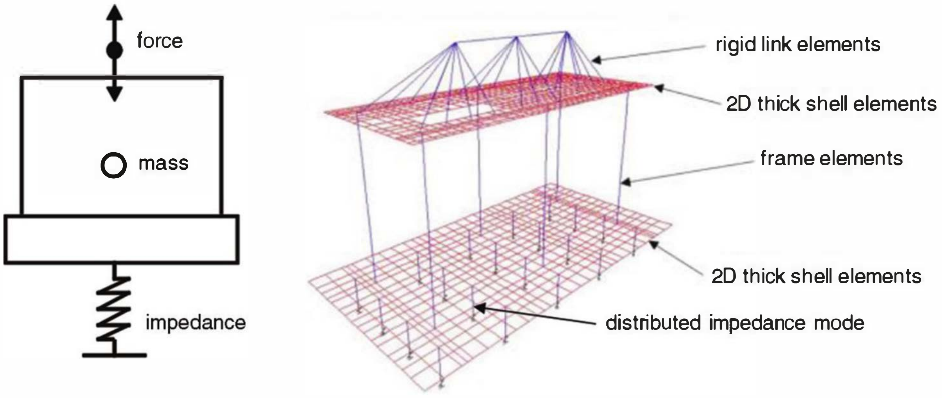
Fig. 6.3.2a—Substructured model.

For a sample application of usage, refer to Example 1 in 6.7.1.

Some finite element analysis software will not allow the input of negative values for stiffness. Soil or pile impedance can be negative under certain circumstances (uniform high Poisson's ratio soil at higher frequencies, pile groups at specific values of spacing and frequency). For such situations, an alternative method is to use a positive stiffness value and to add mass to the foundation. The value of added mass is given by

$$M_\Delta = (K_P + K_N)/\omega_o^2 \quad (6.3.2i)$$

6.3.3 *Vibration isolation*—Impedance characteristics are typically determined from the soil properties, and for tabletop foundations, from the soil properties and the structure flexibility. The designer can modify stiffness and



(a) Lumped Impedance (b) Distributed Impedance on a Pile Foundation

Fig. 6.3.2b—Lumped and distributed soil impedance.

damping values by specifying ground modifications and by changing the proportioning of foundation and structural members/elements. The use of vibration isolation devices provides the ability to establish more precise and stable impedance characteristics, and can capture the benefit of effectively reducing the foundation vibration level.

Vibration isolators are proprietary devices consisting of springs, dampers, or both. Springs are used to reduce the stiffness otherwise provided by the ground/structure. Dampers are used to increase the energy dissipation in the machine-foundation system. These devices can be placed between an inertia block and supporting structural members/elements, as indicated in Fig. 6.3.3a. They can also be placed between a tabletop slab and columns as indicated in Fig. 3.3.4 and 3.3.5.

Isolation systems are used for two purposes. The first purpose is to reduce the transmission of vibrations between a supported machine and the surrounding environment. Isolators could be used with dynamic machinery, such as forging hammers, to prevent machine-induced vibrations from interfering with nearby machines and with worker comfort. The second purpose is to create a less expensive machine support when the cost of installing isolators is less than the cost of the larger structural members/elements that would be required without isolators.

There are three basic spring isolator types used for dynamic machinery applications: rubber (or elastomeric materials) pads, steel springs, and air mounts. Rubber pads are generally the stiffest designs and have a nonlinear load-deflection relationship. Steel springs are less stiff, providing better isolation but low damping. Air-mount types are the least-stiff type of isolator.

Most rubbers and steel springs do not have high level of damping. These designs can be augmented with viscous dampers, providing higher damping levels up to 40 percent, depending on the specific application needs.

The effectiveness of vibration isolators is expressed in two factors: the transmissibility factor and degree of isolation.

The transmissibility factor V_F is the peak force transmitted to the foundation, F_K, divided by the machine's peak dynamic force F. For rotating and reciprocating machines

Fig. 6.3.3a—Isolator-supported fan foundation (photo courtesy of GERB Vibration Control Systems, Inc.).

producing a harmonic dynamic force, the transmissibility factor derived from the equation of motion is indicated in Fig. 6.3.3b for various values of damping ratio D and tuning ratio η. The tuning ratio is defined as the forcing frequency ω_o divided by the undamped natural frequency ω_n of the machine-inertia block-isolator system. A reduction in the transmissibility occurs for values of η larger than the square root of 2. Stiffness isolation devices typically target η values between 2 to 5. Damping devices are most effective when η is near 1 at resonant condition. Note that in the isolation region for $\eta > \sqrt{2}$, higher damping will result in less isolation. Thus, special care should be taken to select suitable damper devices for optimal design between vibration isolation and vibration mitigation near resonance.

The equation for the transmissibility factor, V_F is expressed as

$$V_F = \left|\frac{F_K}{F}\right| = \left|\sqrt{\frac{1+(2D\eta)^2}{(1-\eta^2)^2+(2D\eta)^2}}\right| \quad (6.3.3a)$$

The isolation efficiency as a percentage, I, is derived from the transmissibility factor as

$$I = 100(1 - V_F) \quad (6.3.3b)$$

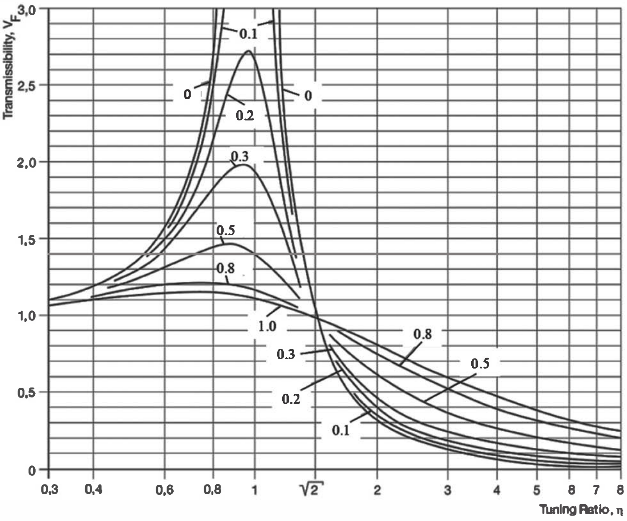


Fig. 6.3.3b—Transmissibility factors by damping ratio D.

or, for $\eta > \sqrt{2}$ and neglecting damping ($D = 0$), the degree of isolation can be expressed as

$$I = 100(\eta^2 - 2)/(\eta^2 - 1) \quad (6.3.3c)$$

For different tuning ratios up to $\eta = 5$, typical for practical applications, the following gives the degree of isolation according to Eq. (6.9):

η	√2	2	3	4	5
I, %	0	66.7	87.5	93.3	95.8

Note that the transmissibility peaks at resonance ($\eta = 1$). When the tuning ratio equals $\sqrt{2}$, transmissibility will once again drop to 1. This is known as the crossover frequency, and the frequencies below this frequency are known as the amplification region. Above this frequency lies the isolation region, where transmissibility is less than 1. Typically, the isolator designer designs a mounting system that puts the primary operating frequencies of the system in the isolation region, frequently at isolation efficiencies greater than 90 percent. Some machine systems need to operate at numerous primary frequencies or frequently go through a startup or slowdown as part of the operation cycle. For these systems, damping in the mount becomes increasingly important when it functions at or near resonance.

6.3.3.1 *Direct support systems*—With direct isolator support of the equipment, the equipment and isolators form the complete dynamic system. The foundation supporting the isolators is subject only to static loads, with the isolators greatly reducing the dynamic force component.

6.3.3.2 *Inertia block systems*—Inertia block systems are used where the mass of the equipment is insufficient to limit the vibration response of the system to acceptable levels. The added mass of the interia block also allows the use of stiffer springs while achieving a low natural frequency of the spring/mass system. For both direct support systems and inertia block systems, the same basic principles in laying out the isolators apply. For a sample calculation, refer to Example 2 in 6.7.2.

6.3.3.3 *Modeling and design*—The arrangement and sizing of isolators should be provided in a manner that properly stabilizes the equipment, distributing both stiffness and damping. The isolator manufacturer should be consulted to provide design aids, design recommendations, and possibly could provide the dynamic calculation for the support if provided with sufficient information. If a design is prepared by the engineer, then the isolator manufacturer's concurrence on the resulting design should be obtained.

Dynamic calculations using vibration isolators are the same as those made for soil- or pile-supported systems, except with the stiffness and damping coming from the isolators. The design of the inertia block follows concrete design recommendations established herein.

If the inertia block is sufficiently thick to be considered rigid (judgment is required), then the dynamic calculation follows that for a soil- or pile-supported rigid block foundation. Resulting natural frequencies of the rigid system should be below the machine's operating frequency (preferably $\eta > 1.4$).

If the inertia block is not considered rigid, then a finite element analysis can be used to model the desired system. Appropriate plate/shell or solid finite elements can be used to represent the inertia block (Fig. 6.3.3.3), rigid elements are generally used to represent the machine, and single joint-support spring elements can be used to represent the isolation devices. Modal frequencies representing rigid body movement of the inertia block should be below the machine's operating frequency. Frequencies representing flexing of the inertia block should generally be above the machine's operating frequency.

If it becomes necessary to determine amplitudes on nearby machines or work areas, then the model should be expanded accordingly. Appropriate plate/shell or solid finite elements may be used to represent the surroundings and two joint link elements can be used to represent the isolation devices. A link element is a member where the user can explicitly input the stiffness and damping. A one-joint link element is like a support point. A two-joint link is like a beam.

6.4—Solution methods

Several methods to solve the equation of motion are commonly applied, depending on the type of loading and the model.

a) **One and two degree-of-freedom (DOF) models**—A simplification of a rigid foundation into one and two DOF models is significant because the solution of the equations of motion can be performed manually or with the aid of a spreadsheet.

b) **Six DOF models**—Solving the six coupled equations of motion for a general block foundation is too lengthy for a purely manual calculation. Computer-aided manual solutions using solvers built into the spreadsheet or engineering calculation software can be used, or a solution can be quickly obtained using commercial vibration analysis software.

Fig. 6.3.3.3—Finite element model of inertia block and machine (image courtesy of GERB Vibration Control Systems, Inc.).

c) **Multi DOF models**—A solution of the equation of motion for flexible foundations will, of necessity, involve finite element analysis computer software. For harmonic loads, a steady-state approach is efficient. A general-purpose approach would be to use a modal time history analysis.

6.4.1 *Modal transformation*—This is a transformation of a coupled multi DOF (MDOF) model into an equivalent system of uncoupled single DOF (SDOF) modes. A model of Q independent degrees of freedom will theoretically result in Q modes. Each mode is then typically solved using the time history method (6.4.2). Finally, the results from each mode are recombined to produce the total system response.

The computational efficiency of a modal transformation results from the concept that not all modes need to be transformed to obtain an accurate result. Additionally, an advantageously intermediate result of the transformation is the frequency (or eigenvalue) and shape (or eigenvector) of each mode. Unlike the more commonly used seismic response spectrum analysis, the forces for a machine dynamic analysis are known, and the magnitude and direction of amplitude results from each mode can be accurately combined.

The efficiency of a modal analysis is improved by determining the least number of modes to reduce computation time while still achieving an accurate result. The criterion to select the number of modes to be analyzed depends on two factors. First, to achieve an accurate calculation of amplitude, the total mass participation should be above a stipulated percentage (usually approximately 95 percent). Second, enough modes should be included so that the highest modal natural frequency is greater than the machine frequency plus a margin. This margin can be determined from a sensitivity analysis; however, experience indicates the margin should be approximately 20 percent. For the sample results in Table 6.4.1, with an operating speed of 25 Hz, the maximum modal frequency should be at least the operating speed of 25 Hz times 1.2 or 30.0 Hz. Thus, 15 modes (with an indicated maximum frequency of 42 Hz) would suffice. The minimum mass participation percentage and resonant range are typically taken from project specifications or internal procedures.

Table 6.4.1—MDOF analysis options (comparative time of analysis using typical sample data)

Analysis	Analysis time	Max frequency, Hz	Mass participation, percent
Steady state	3 seconds	NA	NA
Modal T-H (5 modes)	4 seconds	15	47
Modal T-H (15 modes)	5 seconds	42	100
Modal T-H (200 modes)	15 seconds	412	100
Modal T-H (1000 modes)	12 minutes, 37 seconds	1106	100
Direct T-H	53 minutes, 31 seconds	NA	NA

Briefly, the modal transformation begins with modifying Eq. (6.3) for free vibration by setting the damping $C = 0$ and dynamic load, $F(t) = 0$

$$[M]\{\ddot{y}\} + [K]\{y\} = 0 \quad (6.4.1a)$$

Substituting for amplitude and acceleration

$$\{y\} = \{\phi\}e^{i\omega nt} \quad (6.4.1b)$$

and

$$\{\ddot{y}\} = -\omega_n^2\{\phi\}e^{i\omega nt} \quad (6.4.1c)$$

Applying this substitution to Eq. (6.4.1a), and removal of the common exponential term results in

$$[-M\omega_n^2]\{\phi\} + [K]\{\phi\} = 0 \quad (6.4.1d)$$

and rearranging

$$[K - M\omega_n^2]\{\phi\} = 0 \quad (6.4.1e)$$

For a nontrivial solution, $\phi \neq 0$, the determinant of $[K - M\omega_n^2]$ should be equal to zero, or

$$|K - M\omega_n^2| = 0 \quad (6.4.1f)$$

For relatively low DOF models, a direct solution can be manually obtained by expanding Eq. (6.4.1f) to solve for the natural frequencies, and then solving Eq. (6.4.1e) for the relative amplitudes. For a further discussion related to a frequency analysis, refer to 6.5.2. For sample applications, refer to Example 3 (6.7.3) and Example 4 (6.7.4).

For larger DOF models, a direct solution becomes computationally inefficient. Computer-assisted iterative techniques can be used on Eq. (6.4.1a) to simultaneously solve for natural frequencies and mode shapes. These techniques generally proceed from first (lowest) mode onward with computational complexity increasing with each higher mode. To limit computational time, the procedure is usually halted at a user-specified number of modes.

For a detailed description of modal transformations, solution procedures, and the recombination of individual mode results, refer to Biggs (1964) and Bathe (1996).

6.4.2 *Time history method*—The time history solution method is a resolution of the equation of motion at specific increments of time. The method begins by calculating the system mass, damping, and stiffness. Then initial conditions of displacement, velocity, and acceleration are assumed. These initial conditions are typically taken as zero. Values of displacement, velocity, and acceleration are then calculated at the subsequent time increment using one of a variety of available equations.

This method is more flexible than the steady-state method because the applied force need not be harmonic. This allows the time history method to be used for impact type machinery foundations.

The time history method is commonly used with a modal transformation using the SDOF form of the time step equations applied to each mode. As an example, time step equations based on the constant velocity approach (where subscripts denote the time step numbers, S) are as follows.

Initial condition where the system is at rest ($S = 0$):

$$\text{displacement, } y_0 = 0 \quad (6.4.2a)$$

$$\text{velocity, } \dot{y}_0 = 0 \quad (6.4.2b)$$

$$\text{acceleration, } \ddot{y}_0 = F_0/M \quad (6.4.2c)$$

First time step ($S = 1$):

$$y_1 = y_0 + 0.5\ddot{y}_0 T_\Delta^2 \quad (6.4.2d)$$

$$\ddot{y}_1 = [F_1 - Ky_1 - C(y_1 - y_0)/T_\Delta]/M \quad (6.4.2e)$$

$$\dot{y}_1 = (y_1 - y_0)/T_\Delta + \ddot{y}_1 T_\Delta/2 \quad (6.4.2f)$$

Subsequent steps ($S + 1$):

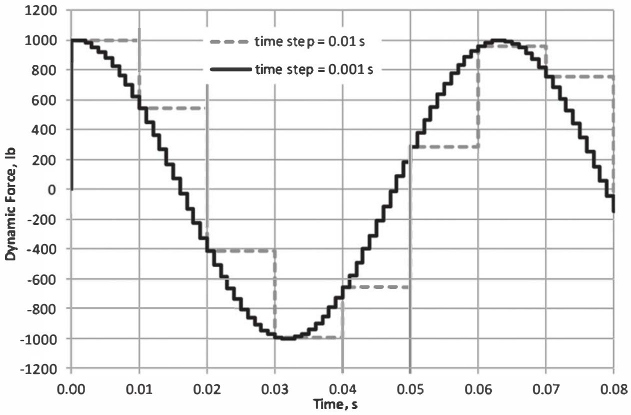


Fig. 6.4.2—Dynamic load versus time step.

$$y_{S+1} = 2y_S - y_{S-1} + \ddot{y}_S T_\Delta^2 \quad (6.2.4g)$$

$$\ddot{y}_{S+1} = [F_{S+1} - Ky_{S+1} - C(y_{S+1} - y_S)/T_\Delta]/M \quad (6.4.2h)$$

$$\dot{y}_{S+1} = (y_{S+1} - y_S)/T_\Delta + \ddot{y}_{S+1} T_\Delta/2 \quad (6.4.2i)$$

In the preceding equations, the mass M and stiffness K are constant with time. The dynamic force is evaluated at each time step and treated as a constant during that time step. Because of this, the user-selected time step increment T_Δ should be short enough to accurately describe the loading function. This effect is illustrated in Fig. 6.4.2.

Because a time history analysis begins at a user-specified initial condition, the transient portion of the response becomes included in the result. For a harmonic load, this implies that there should be enough time steps included to calculate beyond the end of the transient portion of the response. In addition, the deformation result is reported at each time step and the user should manually determine the peak steady-state amplitude. This is normally accomplished by examining a deformation-time plot of the analysis results. For a sample calculation, refer to Example 5 in 6.7.5.

In place of a modal separation, the time step equations can be extended to directly deal with MDOF models. Such a direct time-history analysis can be advantageous for numerous analysis situations; however, it is not commonly used for machine foundations because of the slow progress of the calculation (refer to Table 6.4.1). An additional consideration for a direct time-history analysis is that the type of time step equations to be used becomes significant. For the analysis of machine foundations, an implicit (iterative) type of time step equation would normally apply.

For a detailed description of the time history solution, refer to Biggs (1964) and Bathe (1996).

6.4.3 *Steady-state method*—This is a closed-form equation solution based on the assumption of a continuous harmonic load. The method neglects the (usually not needed) transient portion of the response; however, the method does not directly calculate natural frequencies.

The steady-state method begins by making the following substitutions

$$\{y\} = \{A\}\exp(i\omega_o t) \tag{6.4.3a}$$

$$\{\dot{y}\} = i\omega_o\{A\}\exp(i\omega_o t) \tag{6.4.3b}$$

$$\{\ddot{y}\} = -\omega_o^2\{A\}\exp(i\omega_o t) \tag{6.4.3c}$$

$$\{F(t)\} = \{F_0\}\exp(i\omega_o t) \tag{6.4.3d}$$

Applying this substitution to Eq. (6.3), and removal of the common exponential term results in

$$[-M\omega_o^2]\{A\} + [i\omega_o C]\{A\} + [K]\{A\} = \{F_0\} \tag{6.4.3e}$$

Solving for the amplitude A, the resulting solution is

$$A = \{F_0\}/[K - M\omega_o^2 + iC\omega_o] \tag{6.4.3f}$$

If the damping C is zero, then the amplitude A is a real number representing the peak amplitude Δ and the phase angle θ is zero.

If the damping C is nonzero, then the amplitude A is a complex number (that is, $A = a + ib$). Resolving this complex number into the equivalent polar form yields the desired parameters

$$\text{amplitude, } \Delta = \text{modulus (a, b)} \tag{6.4.3g}$$

$$\text{phase angle, } \theta = \text{argument (a, b)} \tag{6.4.3h}$$

For example applications, refer to Example 6 (6.7.6) and Example 7 (6.7.7).

For a detailed description of the steady-state solution, refer to Biggs (1964) and Kreyszig (1967).

6.5—Frequency analysis

A frequency analysis applies to foundations subjected to harmonic loads and calculates natural frequencies for comparison with specified acceptance criteria. Methods to determine natural frequencies are discussed in 6.5.2 and 6.5.3. Acceptance criterion is discussed in 6.5.1.

The term "natural frequency" is commonly used in acceptance criteria. The term does not stipulate whether the frequency is damped or undamped. The natural frequency is typically assumed to be undamped (especially if a modal transformation is used); however, some practitioners calculate a damped frequency because it more accurately represents actual conditions. For low levels of damping, the undamped and damped natural frequencies are very close. For high levels of damping, the undamped and damped natural frequencies are significantly separated and, if the damping is high enough, there is no resonance. Applicable equations are as follows:

$$\text{undamped natural frequency, } \omega_n = \sqrt{M/K} \tag{6.5a}$$

$$\text{damped natural frequency, } \omega_d = \omega_n\sqrt{1-D^2} \tag{6.5b}$$

$$\text{damping ratio, } D = C/C_{CR} \tag{6.5c}$$

$$\text{critical damping, } C_{CR} = 2\sqrt{MK} \tag{6.5d}$$

Determining the natural frequencies and mode shapes of the machine foundation system provides information about the dynamic characteristic of that system. In addition, calculating the natural frequencies identifies the fundamental frequency, usually the lowest value of the natural frequencies.

Natural frequencies are significant for the following reasons:

a) They indicate the relative degree of stiffness of the machine-foundation-soil system.

b) They can be compared with the frequency of the acting dynamic force so that a possible resonance condition may be prevented. Resonance is prevented when the ratio of the machine operating frequency to the fundamental frequency of the machine-foundation system falls outside the undesirable range (6.5.1). In many cases, six frequencies are calculated consistent with six rigid body motions of the overall machine-foundation system. Each of the six frequencies should be compared with the excitation frequency when checking for resonance condition.

The significance of the mode shapes is as follows:

a) They indicate the deflection pattern that the machine-foundation system assumes when it is left to vibrate after the termination of the disturbing force. Generally, it is the first mode that dominates the vibrating shape, and the higher modes supplement that shape when superimposed.

b) They indicate the relative degree of the structural stiffness among various points of the machine-foundation system—that is, the relationship between different amplitudes and mode shapes. If the flexural characteristics of a foundation are being modeled, a mode shape may indicate that a portion of the foundation (a beam or a pedestal) is relatively flexible at a specific frequency. For a system represented by a six degree-of-freedom (DOF) model, the relative importance of the rocking stiffness and the translational stiffness can be indicated.

c) They can be used as indicators of sensitivity when varying the stiffness, mass, and damping resistance of the machine-foundation system to reduce the vibration amplitudes at critical points. A particular beam or foundation component may or may not be significant to the overall behavior of the system. An understanding of the mode shape can aid in identifying the sensitive components.

Resonance will not be a concern if the natural frequencies are well separated from the machine frequency. If there is a potential for resonance, the engineer should modify the foundation. If resonance is unavoidable, then a more refined calculation can be performed. Refined calculations may include a forced response analysis with a deliberately reduced level of damping. The size and type of equipment play an important role in this decision process.

6.5.1 *Acceptance criteria for frequency*—Acceptance criteria are typically stated in a project specification, supplier requirements, client requirements, published criteria, or internal company criteria.

A specified acceptance criterion is usually expressed in terms of the frequency ratio. The frequency ratio is a term that relates the operating speed of the equipment to the natural frequencies of the foundation. Frequency ratios may be based around either ω_o/ω_n or ω_n/ω_o (operating frequency to natural frequency or its inverse), and engineers or manufacturers should exercise caution to prevent misunderstandings.

Many companies' acceptance criteria require that the natural frequency be a minimum of 20 to 30 percent removed from the operating speed. Some firms have used factors as low as 10 percent or as high as 50 percent.

Frequency ratio is a reasonable design criterion, but one single limiting value does not fit all situations. Where there is greater uncertainty in other design parameters (soil stiffness, for example), more conservatism in the frequency ratio may be appropriate. Similarly, vibration problems can exist even though resonance is not a problem.

There is theoretically one natural frequency for each DOF in a machine-foundation model. For a typical flexible tabletop foundation, there could easily be thousands of DOFs. Because most DOFs are too insignificant to generate significant resonant amplitudes, a minimum mass participation factor is normally specified to determine which modes are significant enough to be evaluated. Mass participation is an indicator of how much mass is mobilized by a particular mode shape.

6.5.2 *Calculation of natural frequencies (direct technique)*—The direct technique determines the undamped natural frequency through a solution derived from the equation of motion. Though the direct technique is fast, it is not normally found in software as a part of a coupled steady-state analysis (6.4.2). However, it is widely applied as a natural step following a MDOF modal transformation (6.4.1). The direct technique is also often used for SDOF models, which are calculated manually.

For a SDOF system (rigid block-type vertical or torsional modes), the solution is obtained by rearranging Eq. (6.4.1f)

$$\omega_n = \text{sqrt}[K_v'/M] \quad (6.5.2a)$$

$$\omega_n = \text{sqrt}[K_\eta'/M_\eta] \quad (6.5.2b)$$

For a two DOF system (rigid block-type coupled horizontal-rocking modes), the following equation is obtained by expanding Eq. (6.4.1f). The solution is obtained by applying a quadratic equation solution

$$\omega_n^4 - [(K_u'/M) + (K_\psi'/M_\psi)]\omega_n^2 + [(K_u')(K_\psi') - (K_C')^2]/(M)(M_\psi) = 0 \quad (6.5.2c)$$

If the soil impedance cannot be considered as linear over the frequency range of interest, then Eq. (6.5.2b) and (6.5.2c) will not automatically yield an accurate result. For nonlinear soil impedance, the assumed frequency value used to determine the impedance may not match the resulting calculated value of natural frequency. This situation can be handled by repeating the calculation of impedance and natural frequency using different values of assumed frequency until the assumed frequency matches the calculated natural frequency. Because of the repetition required, it then becomes more efficient to use the amplitude technique.

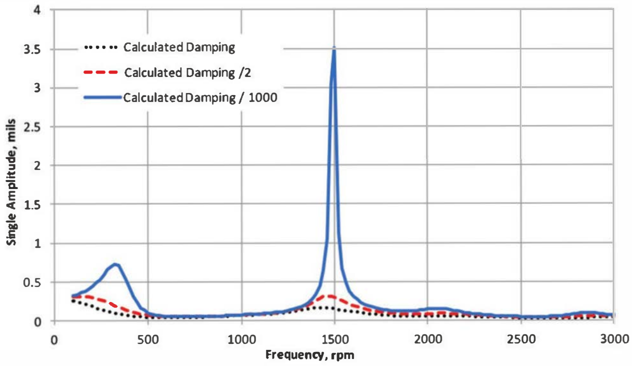


Fig. 6.5.3—Use of reduced damping to detect natural frequencies.

6.5.3 *Calculation of natural frequencies (amplitude technique)*—The amplitude technique determines undamped natural frequency through a steady-state solution of the equation of motion (Eq. (6.4.3f)) by suppressing damping and applying a fictitious dynamic load over a range of frequencies. Peaks in the plotted frequency-amplitude response indicate the desired natural frequency (refer to Fig. 6.5.3). The interpretation of the response curve is typically completed manually.

This method can be used for most any modeling situation, but is best implemented when modeling nonlinear soil impedance (especially pile groups). To obtain undamped natural frequencies, damping values should be greatly suppressed.

This is an indirect method because the natural frequencies should be determined by the user from amplitude results over a range of frequencies. The method may be cumbersome to implement for finite element models for frequency-dependent soil/pile impedance.

6.6—Forced response analysis

In a forced response analysis, the equation of motion (Eq. (6.1) or (6.3)) is solved for the displacements of the machine-foundation system as a function of time. The results of the forced response analysis include displacements, velocities, and accelerations. These results are then compared against allowable limits for acceptance (6.6.1). The allowable limits are typically applicable at discrete locations (for example, the equipment bearing, connections to adjacent equipment, and locations where people may be affected). For flexible foundation MDOF models, additional nodes are commonly included in the analysis model for the locations of interest.

The analysis should account for variation in parameters such as structural stiffness, mass, and soil properties. If amplitude criteria cannot be met, then the foundation size will need to be adjusted.

6.6.1 *Calculation of amplitudes*—The peak amplitude may be calculated by using the methods described in 6.4.

6.6.2 *Acceptance criteria for amplitude*—Acceptance criteria are typically stated in a project specification, supplier requirements, client requirements, published criteria, applicable legally adopted codes, or internal company criteria. The engineer should confirm applicable criteria prior to beginning a design.

The main purposes of the foundation system with respect to dynamic loads include limiting vibrations, internal loads, and stresses within the equipment. The foundation system also limits vibrations in the areas around the equipment where other vibration-sensitive equipment may be installed, where personnel may have to work on a regular basis, or where damage to the surrounding structures may occur. These performance criteria are usually established based on vibration amplitudes at key points on or around the equipment and foundation system. These amplitudes are based on displacement, velocity, or acceleration units. Displacement limitations are commonly based on peak-to-peak amplitudes measured in mils (0.001 in.) or micrometers (10^{-6} m). Velocity limitations are typically either based on peak velocities or root-mean-square (rms) velocities in units of in./s or mm/s. Displacement criteria are almost always frequency-dependent with greater amplitudes tolerated at slower speeds. Velocity criteria may depend on frequency but are often independent. Acceleration criteria may be constant with frequency or may be frequency-dependent.

Some types of equipment operate at a constant speed whereas other types operate across a range of speeds. The engineer should consider the effect of these speed variations during the foundation design.

6.6.2.1 *Machine limits*—The vibration limits applicable to the machine are normally set by the equipment manufacturer or are specified by the equipment operator or owner. The limits are usually predicated on either limiting damage to the equipment or ensuring proper long-term performance of the equipment. Limits specified by operators of the machinery and engineers are usually based on such factors as experience or the installation of additional vibration monitoring equipment.

For rotating equipment (fans, pumps, and turbines), the typical criterion limits the vibration displacements or velocities at the bearings of the rotating shaft. Excessive vibrations of the bearings increase maintenance requirements and lead to premature failure of the bearings. Often, rotating equipment has sensors for monitoring vibrations and control switches to alarm operators or automatically shut down the equipment if vibrations become excessive.

Reciprocating equipment, such as diesel generators and compressors, tends to be more dynamically rugged than rotating equipment. At the same time, reciprocating equipment often generates larger dynamic forces. While the limits may be higher, motions are measured at bearing locations. In addition, operators of reciprocating compressors often monitor vibrations of the compressor base relative to the foundation (sometimes referred to as frame movement) as a measure of the foundation and machine-mounting condition and integrity.

Impulsive machines, such as presses and forging hammers, tend not to have specific vibration limitations that are controllable by the foundation design. With these machines, it is important to recognize the difference between the inertial forces and equipment dynamics as contrasted with the foundation system dynamics. The equipment forces can generate significant accelerations and stresses that are unrelated to the stiffness, mass, or other design aspect of the foundation system. Thus, monitoring accelerations, in particular on an equipment frame, may not be indicative of foundation suitability or adequacy.

Researchers have presented various studies and papers addressing the issues of machinery vibration limits. This variety is reflected in the standards of engineering companies, plant owners, and industry standards. When the equipment manufacturer does not establish limits, recommendation from ISO 10816-1, Blake (1964), and Baxter and Bernhard (1967) are often followed. Most of these studies relate directly to rotating equipment. In many cases, they are also applicable to reciprocating equipment. Rarely do these studies apply to impulsive equipment.

ISO 10816 contains seven parts to address evaluation of machinery vibration by measurements on the nonrotating parts. ISO 10816-1 provides general guidelines and sets the overall rules. Subsequent parts provide specific criteria for specific machinery types. These standards are primarily directed toward in-place measurements for the assessment of machinery operation. They are not intended to identify design standards. Engineers, however, have used predecessor documents to ISO 10816 as a baseline for design calculations and can be expected to do similarly with these more recent standards.

ISO 10816 presents vibration criteria in terms of rms velocity. Where there is complexity in the vibration signal (beyond simple rotor unbalance), the rms velocity basis provides the broad measure of vibration severity and can be correlated to likely machine damage. For situations where the pattern of motion is fairly characterized by a single frequency, such as simple rotor unbalance, the rms, velocities can be multiplied by √2 to determine corresponding peak velocity criteria. For these same cases, displacements (Δ) can be calculated as

$$\Delta = V_{PEAK}/\omega_o = \sqrt{2}\, V_{RMS}/\omega_o \qquad (6.6.2.1)$$

The displacements determined from Eq. (6.6.2.1) are zero-to-peak displacement values, which can be doubled to determine peak-to-peak values. If the motion is not a simple pure harmonic motion, a simple relationship among the rms displacement, rms velocity, peak velocity, zero-to-peak displacement, and peak-to-peak displacement does not exist.

ISO 10816-1 identifies four areas of interest with respect to the magnitude of vibration measured:

a) Zone A: vibration typical of new equipment

b) Zone B: vibration normally considered acceptable for long-term operation

c) Zone C: vibration normally considered unsatisfactory for long-term operation

d) Zone D: vibration normally considered severe enough to damage the machine

The subsequent parts of ISO 10816 establish the boundaries between these zones as applicable to specific equipment. ISO 10816-2 establishes criteria for large, land-based, steam-turbine generator sets rated over 67,000 horsepower (50 MW). ISO 10816-3 is the most general of the standards. It addresses in-place evaluation of general industrial machinery nominally over 15 kW and operating between 120 and 15,000 rpm. Within ISO 10816-3, criteria are established for four different groups of machinery, and provisions include either flexible or rigid support conditions. Criteria are also established based on both rms velocity and rms displacement. ISO 10816-4 identifies evaluation criteria for gas-turbine-driven power generation units (excluding aircraft derivatives) operating between 3000 and 20,000 rpm. ISO 10816-5 applies to machine sets in hydropower facilities and pumping plants. ISO 10816-6 provides evaluation criteria for reciprocating machines with power ratings over 134 horsepower (100 kW). ISO 10816-7 gives guidance for the evaluation of vibration on rotodynamic pumps for industrial applications with nominal power above 1 kW. The scope of Part 5 is not applicable to general equipment foundations and the criteria of Part 6 needs additional verification and is seldom used.

Lifshits et al. (1986) also establishes vibration limitation. This document follows Blake's (1964) approach of identifying five different categories from No Faults to Danger of Immediate Failure. In addition, a series of correction factors are established to broaden the applicability to a wider range of equipment and measurement data.

Blake (1964) has become a common basis for some industries and firms. His work presented a standard vibration chart for process equipment with performance ratings from No Faults (typical of new equipment) to Dangerous (shut it down now to avoid danger). The chart was primarily intended to aid plant personnel in assessing field installations and determining maintenance plans. Service factors for different types of equipment are used to allow widespread use of the basic chart. This tool uses vibration displacement (in. or mm) rather than velocity and covers speed ranges from 100 to 10,000 rpm. Figure 6.2b presents the basic chart established by Blake (1964).

Baxter and Bernhard (1967) offered more general vibration tolerances and is widely referenced. Again, with primary interest to the plant maintenance operations, they established the General Machinery Vibration Severity Chart, shown in Fig. 5.8.1a, with severity ranging from extremely smooth to very rough. These are plotted as displacement versus vibration frequency so that the various categories are differentiated along lines of constant peak velocity.

The American Petroleum Institute (API) also has a series of standards for equipment common in the petrochemical industry (API 541, 610, 612, 613, 617, 618, and 619). ISO 10816-3 can be applied for some large electrical motors. HI/ANSI 9.6.4 can be applied for water pumps.

Internal company practices frequently provide the basis to be used for design. Figure 5.8.1b(a) and 5.8.1b(b)c provides a comparison of five company standards, designated A through E, plotted against the Baxter and Bernhard (1967) background. Permissible amplitudes are presented for reciprocating (recip) and for rotating (rot) machines.

Most of the permissible values shown in Fig. 5.8.1b(a) and 5.8.1b(b) tend to decrease with increasing frequency. The simplest example of this is Companies D and E, which use PIP STC01015. PIP STC01015 specifies a constant velocity that results in the straight sloping lines shown in Fig. 5.8.1b. Companies A and B use a constant value of amplitude for reciprocating machines. The collection of amplitudes shown in the figure indicates a significant variation at low frequencies, with Companies A and B at the low end of the range and Companies D and E at the high end. All values are fairly close, at frequencies just above 1000 rpm. Harris (1996) contains further general information.

6.6.2.2 *Physiological limits*—Human perception and sensitivity to vibration is ambiguous and subjective. Researchers have studied and investigated this topic, but there are no clear uniform U.S. standards. Important issues are the personnel expectations and needs and the surrounding environment. Industrial workers near such equipment have different expectations than office workers or the public.

ISO 2631, Parts 1, 2, 4, and 5, provides guidance for human exposure to whole-body vibration and considers different comfort levels and duration of exposure. The frequency of the accelerations also impact fatigue and proficiency. Industrial workers near such equipment have different expectations than office workers or the public.

The modified Reiher-Meister figure (barely perceptible, noticeable, and troublesome) is also used to establish limits with respect to personnel sensitivity, shown in Fig. 6.6.2.2.

DIN 4150-3 defines permissible velocities suitable for assessment of short-term vibrations on structures, which are given in Table 6.6.2.2. Furthermore, DIN 4150-2 defines limitations for allowable vibrations based on perception as a function of location (residential, light industrial) and either daytime or nighttime. In the design of foundations for dynamic equipment, most engineering offices do not consider human perception to vibrations, unless there are extenuating circumstances, such as proximity to office or residential areas.

There are no conclusive limitations on the effects of vibration of surrounding buildings. Figure 6.6.2.2 identifies levels of vibration from mining operations that have damaged structures.

6.7—Sample calculations

The following examples are provided to illustrate implementation of the procedures described in Chapter 6 and to illustrate the use of a manual calculation.

6.7.1 *Example 1: Distribution of lumped impedance on a finite element method mat foundation*—This example illustrates how a lumped value of impedance can be approximately separated into distributed values for input into a finite element model, as described in 6.3.2.

Given: lumped impedance results at the center of resistance as follows:

$K_v = 2.604 \times 10^6$ kip/ft	$C_v = 1.480 \times 10^4$ kip-s/ft
$K_u = 8.400 \times 10^5$ kip/ft	$C_u = 5.400 \times 10^3$ kip-s/ft
$K_\psi = 3.790 \times 10^{11}$ kip-ft/rad	$C_\psi = 8.010 \times 10^5$ kip-ft-s/rad
$K_c = 5.418 \times 10^9$ kip/rad	$C_c = 2.170 \times 10^4$ kip-s/rad

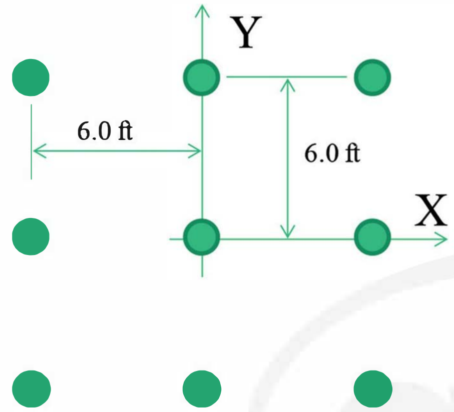
Given: layout of mat foundation elements with nodes at locations as follows:

Table 6.6.2.2—Short-term permissible values (DIN 4150-3)

Type of building	Foundation (1 to 10 Hz)	Foundation (10 to 50 Hz)	Foundation (50 to 100 Hz)	Top complete floor (all frequencies)
Industrial and commercial	0.8 in./s (20 mm/s)	0.8 to 1.6 in./s (20 to 40 mm/s)	1.6 to 2.0 in./s (40 to 50 mm/s)	1.6 in./s (40 mm/s)
Residential	0.2 in./s (5 mm/s)	0.2 to 0.6 in./s (5 to 15 mm/s)	0.6 to 0.8 in./s (15 to 20 mm/s)	0.6 in/s (15 mm/s)
Special or sensitive	0.1 in./s (3 mm/s)	0.1 to 0.3 in./s (3 to 8 mm/s)	0.3 to 0.4 in./s (8 to 10 mm/s)	0.3 in./s (8 mm/s)

Number of nodes, $N = 9$
Spacing of nodes, 3 x 3 pattern, 6 ft spacing
Pattern parameter:

$\sum L_P^2$ = (3 each)(–6 ft)^2 + (3 each)(0 ft)^2 + (3 each)(6 ft)^2 = 216 ft^2



Single-node vertical stiffness using Eq. (6.3.2a):

$k_v = K_v/N$ = (2.604 × 10^6 kip/ft)/(9 each) = 289,333 kip/ft = 4.22 × 10^6 kN/m

Single-node horizontal stiffness using Eq. (6.3.2b):

$k_u = K_u/N$ = (8.400 × 10^5 kip/ft)/(9 each) = 93,333 kip/ft = 1.36 × 10^6 kN/m

Single-node rocking stiffness using Eq. (6.3.2c):

$k_\psi = [K_\psi - k_v(\sum L_P^2)]/N$
= [(3.790 × 10^{11} kip-ft/rad) – (2.893 × 10^5 kip/ft)(216 ft^2)]/(9 each) = 4.210 × 10^{10} kip-ft/rad = 5.71 × 10^{11} kN-m/rad

Single-node cross stiffness using Eq. (6.3.2d):

$k_c = K_c/N$ = (5.418 × 10^9 kip/rad)/(9 each) = 6.020 × 10^8 kip/rad = 2.68 × 10^9 kN/rad

Single-node vertical damping using Eq. (6.3.2e):

$c_v = C_v/N$ = (1.480 × 10^4 kip-s/ft)/(9 each) = 1644 kip-s/ft

Single-node horizontal damping using Eq. (6.3.2f):

$c_u = C_u/N$ = (5.400 × 10^3 kip-s/ft)/(9 each) = 600 kip-s/ft = 8756.33 kN-s/m

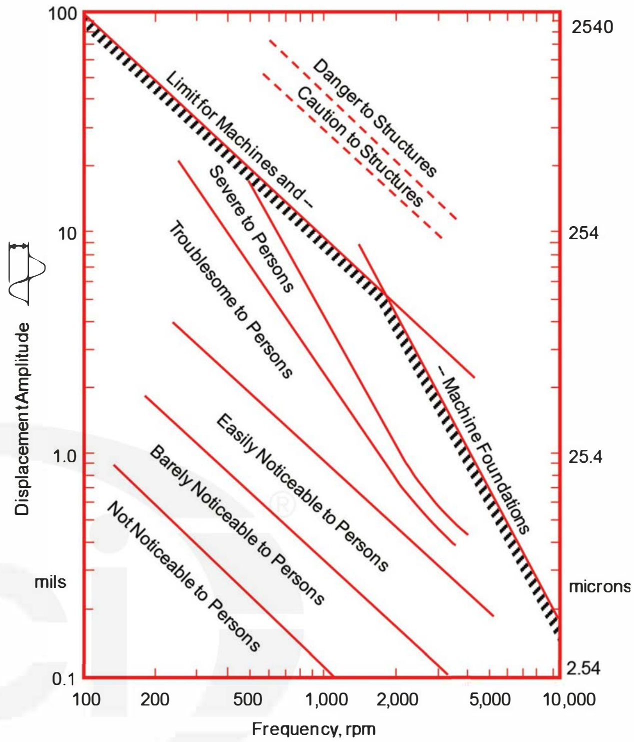


Fig. 6.6.2.2—Reiher-Meister chart (Richart et al. 1970).

Single-node rocking damping using Eq. (6.3.2g):

$c_\psi = [C_\psi - c_v(\sum L_P^2)]/N$ = [(8.010 × 10^5 kip-ft-s/rad) – (1.644 × 10^3 kip-s/ft)(216 ft^2)]/(9 each) = 49,544 kip-ft/rad = 67172.64 kN-m/rad

Single-node cross damping using Eq. (6.3.2h):

$c_c = C_c/N$ = (2.170 × 10^4 kip-s/rad)/(9 each) = 2411 kip-s/rad = 10724.66 kN-s/rad

6.7.2 *Example 2: Amplitude determination using vibration isolation*—This example illustrates a dynamic calculation for a fan supported on an inertia block with vibration isolators (Fig. 6.3.3a). This example is limited to the vertical mode. For an actual design, all six modes would need to be considered.

Fan data:
Machine size: 2.4 m wide by 5.5 m long
Machine speed: ω_o = 78.5 rad/s (750 rpm)
Machine mass: 5500 kg
Peak dynamic force: 4600 N

Permissible peak-to-peak amplitude at foundation: 75 μm
Permissible tuning ratio: ω_o/ω_n = outside of 0.6 to 1.4 resonant range

Calculation:
Assume no damping and a rigid inertia block
Select an inertia block: 2.7 m wide by 5.8 m long by 0.46 m thick
Inertia block mass: (2.7 x 5.8 x 0.46 m)(2400 kg/m^3) = 17,289 kg
In this example, springs are selected, by trial calculation, that target a natural frequency of one-third that of the fan.
Select springs: Eight each at 19,600 N/cm
Stiffness:

K_v = (8 each)(19,600 N/cm)(100 cm/m) = 15,680,000 N/m

Mass:

M = 5500 kg + 17,289 kg = 22,789 kg

Natural frequency using Eq. (6.5a):

ω_n = sqrt(K_v/M) = sqrt[(15,680,000 N/m)/(22,789 kg)]
= 26.2 rad/s

Tuning ratio:

$\eta = \omega_o/\omega_n$ = (78.5 rad/s)/(26.2 rad/s) = 3 > 1.4 **OK**

Degree of isolation from Eq. (6.3.3c):

$I = 100(\eta^2 - 2)/(\eta^2 - 1) = 100(3^2 - 2)/(3^2 - 1) = 87.5\%$

Transmissibility factor using Eq. (6.3.3b) rearranged:

V_F = 100% – 87.5% = 12.5%

Single (zero-to-peak) displacement amplitude:

$SA = F \times V_F/\text{abs}(K_v - M\omega_o^2)$
= (4600 N) × 0.125/abs[(15,680,000 N/m) – (22,789 kg)(78.5 rad/s)^2] = 4.61 × 10^{-6} m = 4.61 μm = 0.181 mil

Double (peak-to-peak) amplitude:

DA = 2(SA) = 2(4.61 μm) = 9.22 μm
= 0.363 << 75 μm = 2.95 **OK**

Note that nonrigid inertia blocks would require the additional calculation of natural frequencies within the block itself.

6.7.3 *Example 3: SDOF determination of natural frequency*—This example illustrates a manual calculation for vertical translation as described in 6.5.2.
Stiffness: K_V' = 11,000 lbf/ft
Weight: WT = 3000 lbf
Mass: (WT)/g = (3000 lbf)/(32.2 ft/s^2) = 93 lbf-s^2/ft

Calculate the natural undamped frequency using Eq. (6.5.2a):

ω_n = sqrt[K_v'/M] = sqrt[(11,000 lbf/ft)/(93 lbf-s^2/ft)]
= 10.88 rad/s

6.7.4 *Example 4: Two DOF determination of natural frequency using modal transformation*—This example illustrates a manual calculation for coupled horizontal translation and rocking as described in 6.4.1.

Given impedance at center of resistance, aligned vertically with center of gravity:
$K_u = 8.518 \times 10^7$ lbf/ft
$K_\psi = 5.085 \times 10^9$ lbf-ft/rad
K_c = 0 lbf/rad; supposedly a soil-supported mat foundation
Mass: M = 5350 lbf-s^2/ft
Mass moment of inertia: M_ψ = 240,000 lbf-s^2-ft
Base to center of gravity: h = 3.861 ft

Calculate stiffness at center of gravity:
$K_u' = K_u = 8.518 \times 10^7$ lbf/ft
$K_\psi' = K_\psi + K_u h^2 - 2K_c h$ = (5.085 × 10^9 lbf-ft/rad) + (8.518 × 10^7 lbf/ft)(3.861 ft)^2 – 2(0 lbf/rad)(3.861 ft)
= 6.355 × 10^9 lbf-ft/rad
$K_c' = K_c - K_u h$ = (0 lbf/rad) – (8.518 × 10^7 lbf/ft)(3.861 ft)
= –3.289 × 10^8 lbf/rad

Quadratic solution of Eq. (6.5b):
$B_1 = -(K_u'/M) - (K_\psi'/M_\psi)$
= –(8.518 × 10^7 lbf/ft)/(5350 lbf-s^2/ft)
– (6.355 × 10^9 lbf-ft/rad)/(240,000 lbf-s^2-ft) = –4.238 × 10^4 s^{-2}
$B_2 = [(K_u')(K_\psi') - (K_c')^2]/(M)(M_\psi)$
= [(8.518 × 10^7 lbf/ft)(6.355 × 10^9 lbf-ft/rad)
– (–3.289 × 10^8 lbf/rad)^2]/(5350 lbf-s^2/ft)(240,000 lbf-s^2-ft)
= 3.370 × 10^8 s^{-4}

$$\omega_{n1}^2 = \{-B_1 + \sqrt{B_1^2 - 4B_2}\}/2$$
$$= \left\{\begin{array}{l} -(-4.238\times10^4 \ 1/\text{s}^2) \\ \hline +\sqrt{(-4.238\times10^4 \ 1/\text{s}^2)^2 - 4(3.370\times10^8 \ 1/\text{s}^4)} \end{array}\right\}/2$$
$$= 31,775 \ \text{s}^{-2}$$
ω_{n1} = 178 rad/s
$$\omega_{n2}^2 = \{-B_1 - \sqrt{B_1^2 - 4B_2}\}/2$$
$$= \left\{\begin{array}{l} -(-4.238\times10^4 \ 1/\text{s}^2) \\ \hline -\sqrt{(-4.238\times10^4 \ 1/\text{s}^2)^2 - 4(3.370\times10^8 \ 1/\text{s}^4)} \end{array}\right\}/2$$
$$= 10,605 \ \text{s}^{-2}$$
ω_{n2} = 103 rad/s

For the first mode, the shape factors derived from Eq. (6.4.1d) are:
$\phi_1 = -K_c'/(K_u' - M\omega_{n1}^2)$ = –(–3.289 × 10^8 lbf/rad)/[(8.518 × 10^7 lbf/ft) – (5350 lbf-s^2/ft)(178 rad/s)^2] = –3.87
ϕ_2 = 1.0

For the second mode, the shape factors derived from Eq. (6.4.1d) are:

$\phi_1 = -K_c'/(K_u' - M\omega_{n2}^2) = -(-3.289 \times 10^8$ lbf/rad)/ $[(8.518 \times 10^7$ lbf/ft) – (5350 lbf-s²/ft)(103 rad/s)²] = 11.58
$\phi_2 = 1.0$

6.7.5 *Example 5: SDOF determination of amplitude using time-history method*—This example illustrates a manual calculation for peak vertical translation as described in 6.4.2.
Magnitude of force: F_V = 1000 lbf
Frequency of force: ω_o = 314.16 rad/s (3000 rpm)
Mass: M = 2960 lbf-s²/ft
Stiffness: K_v' = 135,300,000 lbf/ft
Damping coefficient: C_v' = 36,840 lbf-s/ft
Select time step (T_Δ):
cycle time = (1/3000 min/rev)(60 s/min) = 0.02 s
Time step: T_Δ = (cycle time)/10 = (0.02 s)/10 = 0.002 s

Calculate the initial conditions (S = 0) using Eq. (6.4.2a) through (6.4.2c):
$F_0 = F_V\cos(\omega_o t)$ = (1000 lbf)cos(314.16 rad/s × 0 s) = 1000 lbf
$y_0 = 0$
$\dot{y}_0 = 0$
$\ddot{y}_0 = F_0/M_V$ = (1000 lbf)/(2960 lbf-s²/ft) = 0.338 ft/s²

Calculate the first time step (S = 1) using Eq. (6.4.2d) through (6.4.2f):
$t_1 = 0 + T_\Delta$ = 0.002 s
$F_1 = F_V\cos(\omega_o t_1)$ = (1000 lbf)cos(314.16 rad/s × 0.002 s) = 809 lbf
$y_1 = y_0 + 0.5\ddot{y}_0 T_\Delta^2 = 0 + 0.5(0.338$ ft/s²)(0.002 s)² = 6.76×10^{-7} ft
$\ddot{y}_1 = [F_1 - Ky_1 - C(y_1 - y_0)/T_\Delta]/M$ = [(809 lbf) – (135,300,000 lbf/ft)(6.76×10^{-7} ft) – (36,840 lbf-s/ft)(6.76×10^{-7} – 0 ft) / (0.002 s)]/(2960 lbf-s²/ft) = 0.238 ft/s²
$\dot{y}_1 = (y_1 - y_0)/T_\Delta + \ddot{y}_1 T_\Delta/2$ = (6.76×10^{-7} – 0 ft)/0.002 s + (0.238 ft/s²)(0.002 s)/2 = 5.76×10^{-4} ft/s

Calculate the second time step (S = 2) using Eq. (6.4.2g) through (6.4.2i):
$t_2 = t_1 + T_\Delta$ = (0.002 s) + (0.002 s) = 0.004 s
$F_2 = F_V\cos(\omega_o t_2)$ = (1000 lbf)cos(314.16 rad/s × 0.004 s) = 309 lbf
$y_2 = 2y_1 - y_0 + \ddot{y}_1 T_\Delta^2$ = 2(6.76×10^{-7} ft) – (0) + (0.238 ft/s2)(0.002 s)² = 2.30×10^{-6} ft
$\ddot{y}_2 = [F_2 - Ky_2 - C(y_2 - y_1)/T_\Delta]/M$ = [(309 lbf) – (135,300,000 lbf/ft)(2.30×10^{-6} ft) – (36,840 lbf-s/ft)(2.30×10^{-6} – 6.76×10^{-7} ft)/(0.002 s)]/(2960 lbf-s²/ft) = –0.011 ft/s²
$\dot{y}_2 = (Y_2 - Y_1)/T_\Delta + \ddot{Y}_2 T_\Delta/2$ = (2.30×10^{-6} – 6.76×10^{-7} ft)/(0.002 s) + (–0.011 ft/s²)(0.002 s)/2 = 8.01×10^{-4} ft/s

The repeated calculation is typically performed on a spreadsheet as follows:

Time, s	Dynamic load, lbf	Deflection, ft	Velocity, ft/s	Accelera-tion, ft/s²
0.0000	1000	0.000	0.000	0.338
0.0020	809	6.757×10^{-07}	5.731×10^{-04}	0.235
0.0040	309	2.293×10^{-06}	7.981×10^{-04}	–0.010
0.0060	–309	3.868×10^{-06}	5.003×10^{-04}	–0.287
0.0080	–809	4.292×10^{-06}	-2.535×10^{-04}	–0.466
0.0100	–1000	2.854×10^{-06}	-1.174×10^{-03}	–0.454
0.0120	–809	-4.007×10^{-07}	-1.859×10^{-03}	–0.232
0.0140	–309	-4.583×10^{-06}	-1.962×10^{-03}	0.129
0.0160	309	-8.247×10^{-06}	-1.334×10^{-03}	0.498
0.0180	809	-9.919×10^{-06}	-1.081×10^{-04}	0.728

Time, s	Dynamic load, kN	Deflection, µm	Velocity, mm/s	Accelera-tion, m/s²
0.0000	4.448	0.000	0.000	0.103
0.0020	3.599	0.206	0.174	0.072
0.0040	1.375	0.699	0.243	–0.003
0.0060	–1.375	1.179	0.152	–0.087
0.0080	–3.599	1.308	–0.077	–0.142
0.0100	–4.448	0.870	–0.358	–0.138
0.0120	–809	–0.122	–0.567	–0.071
0.0140	–309	–1.397	–0.598	0.039
0.0160	309	–2.514	–0.407	0.152
0.0180	809	–3.023	–0.033	0.222

The resulting deformation time plot is as follows in Fig. 6.7.5.

The steady-state peak amplitude is read from the spreadsheet at a time period after the observable transient response has subsided.

$$\text{single amplitude (at } t = 1.8700\text{ s)} = 6.749 \times 10^{-6}\text{ ft} = 0.08\text{ mils } (2\ \mu\text{m})$$

The use of a shorter time step would result in an amplitude that matches the corresponding result in Example 6 (6.7.6).

6.7.6 *Example 6: SDOF determination of response using steady-state method*—This example illustrates a manual calculation for the peak vertical translation response as described in 6.4.3 without the use of complex arithmetic.

Magnitude of force: F_V = 1000 lbf
Frequency of force: ω_o = 314.16 rad/s (3000 rpm)
Mass: M = 2960 lbf-s²/ft
Stiffness: K_v = 135,300,000 lbf/ft
Damping coefficient: C_v = 36,840 lbf-s/ft

Calculate parameters based on Eq. (6.4.3f):
$a = K_v - M\omega_o^2$ = (135,300,000 lbf/ft) – (2960 lbf-s²/ft)(314.16 rad/s)² = -1.565×10^8 lbf/ft
$b = -C_v\omega_o$ = –(36,840 lbf-s/ft)(314.16 rad/s) = -1.157×10^7 lbf/ft

Calculate the peak amplitude:
$\Delta_v = F/\text{sqrt}[a^2 + b^2]$ = (1000 lbf)/sqrt[(-1.565×10^8 lbf/ft)² + (-1.157×10^7 lbf/ft)²] = 6.358×10^{-6} ft = 0.076 mils = 1.93 µm

Calculate the phase angle (with both a and b negative):
$\theta_v = 180 - \arctan[b/a] = 180 - \arctan[(-1.157 \times 10^7$ lbf/ft)/(-1.565×10^8 lbf/ft)] = –175.779 deg

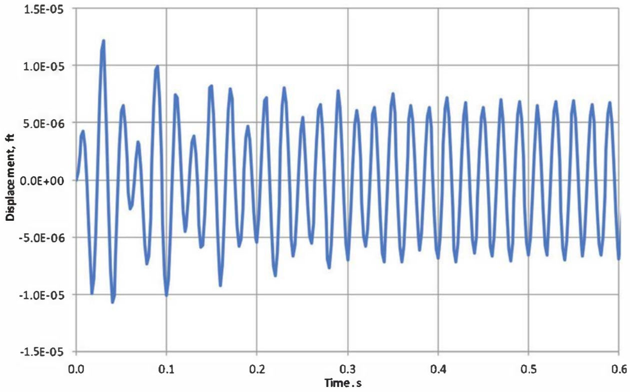


Fig. 6.7.5—Time-history amplitude plot.

6.7.7 *Example 7: Two DOF determination of response using steady state method*—This example illustrates a manual calculation for peak horizontal translation and rocking response as described in 6.4.3.

Given impedance at center of resistance:

$K_u = 5.634 \times 10^7$ lbf/ft	$C_u = 1.441 \times 10^5$ lbf-s/ft
$K_\psi = 3.636 \times 10^9$ lbf-ft/rad	$C_\psi = 7.225 \times 10^6$ lbf-ft-s/rad
$K_c = -4.772 \times 10^7$ lbf/rad	$C_c = -6.395 \times 10^4$ lbf-s/rad

Given inertia about center of gravity:

$M = 2956$ lbf-s²/ft	$M_\psi = 62{,}183$ lbf-s²-ft

Given height from center of resistance to center of gravity:

$h = 3.418$ ft

Given machine properties:

$\omega_o = 314$ rad/s (3000 rpm)

Force: $F_u = 1000$ lbf

Moment: $F_\psi = 0$ lbf-ft

The following equations are from Fang (1991), and modified to include damping:

$B_1 = (K_u) - (M)(\omega_o)^2 + i(C_H)(\omega_o) = (5.634 \times 10^7 \text{ lbf/ft}) - (2956 \text{ lbf-s}^2\text{/ft})(314 \text{ rad/s})^2 + i(1.441 \times 10^5 \text{ lbf-s/ft})(314 \text{ rad/s})$
$= -2.354 \times 10^8 + i4.527 \times 10^7$ lbf/ft

$B_2 = (K_c) - (K_u)(h) + i(C_c)(\omega_o) - i(C_u)(h)(\omega_o)$
$= (-4.772 \times 10^7 \text{ lbf/rad}) - (5.634 \text{ x } 10^7 \text{ lbf/ft})(3.418 \text{ ft}) + i(-6.395 \times 10^4 \text{ lbf-s/rad})(314 \text{ rad/s}) - i(1.441 \text{ x } 10^5 \text{ lbf-s/ft})(3.418 \text{ ft})(314 \text{ rad/s}) = -2.403 \times 10^8 - i1.748 \times 10^8$ lbf/rad

$B_3 = (K_\psi) - (M_\psi)(\omega_o)^2 + (K_u)(h)^2 - (2)(K_c)(h) + i(C_\psi)(\omega_o) + i(C_u)(\omega_o)(h)^2 - i(2)(C_c)(\omega_o)(h) = (3.636 \times 10^9 \text{ lbf-ft/rad}) - (62{,}183 \text{ lbf-s}^2\text{-ft})(314 \text{ rad/s})^2 + (5.634 \times 10^7 \text{ lbf/ft})(3.418 \text{ ft})^2 - (2)(-4.772 \times 10^7 \text{ lbf/rad})(3.418 \text{ ft}) + i(7.225 \times 10^6 \text{ lbf-ft-s/rad})(314 \text{ rad/s}) + i(1.441 \times 10^5 \text{ lbf-s/ft})(314 \text{ rad/s})(3.418 \text{ ft})^2 - i(2)(-6.395 \times 10^4 \text{ lbf-s/rad})(314 \text{ rad/s})(3.418 \text{ ft})$
$= -1.517 \times 10^9 + i2.936 \times 10^9$ lbf-ft/rad

$B_4 = (B_1)(B_3) - (B_2)^2 = (-2.354 \times 10^8 + i4.527 \times 10^7 \text{ lbf/ft})(-1.517 \times 10^9 + i2.936 \text{ x } 10^9 \text{ lbf-ft/rad}) - (-2.403 \times 10^8 - i1.748 \times 10^8 \text{ lbf/rad})^2 = 1.970 \times 10^{17} - i8.440 \times 10^{17} \text{ lbf}^2$

$A_u = [(B_3)(F_u) - (B_2)(F_\psi)]/B_4 = [(-1.517 \times 10^9 + i2.936 \times 10^9 \text{ lbf-ft/rad})(1000 \text{ lbf}) - (-2.403 \times 10^8 - i1.748 \times 10^8 \text{ lbf/rad})(0 \text{ lbf-ft})]/(1.970 \times 10^{17} - i8.440 \times 10^{17} \text{ lbf}^2)$
$= -3.697 \times 10^{-6} - i9.341 \times 10^{-7}$ ft

Amplitude:

$\Delta_u = \text{modulus } (A_u) = \text{sqrt}[a^2 + b^2] = \text{sqrt}[(-3.697 \times 10^{-6} \text{ ft})^2 + (-9.341 \times 10^{-7} \text{ ft})^2] = 3.81 \times 10^{-6}$ ft

Phase angle:

$\theta_u = \text{argument } (A_u)$, with both a and b being negative:
$= -180 + \arctan[b/a] = -180 + \arctan[(-9.341 \times 10^{-7} \text{ ft})/(-3.697 \times 10^{-6} \text{ ft})] = -165.8$ deg

$A_\psi = [(B_1)(F_\psi) - (B_2)(F_u)]/B_4 = [(-2.354 \times 10^8 + i4.527 \times 10^7 \text{ lbf/ft})(0 \text{ lbf-ft}) - (-2.403 \times 10^8 - i1.748 \times 10^8 \text{ lbf/rad})(1000 \text{ lbf})]/(1.970 \times 10^{17} - i8.440 \times 10^{17} \text{ lbf}^2)$
$= -1.334 \times 10^{-7} + i3.159 \times 10^{-7}$ ft

Amplitude:

$\Delta_\psi = \text{modulus } (A_\psi) = \text{sqrt}[a^2 + b^2] = \text{sqrt}[(-1.334 \times 10^{-7} \text{ ft})^2 + (3.159 \times 10^{-7} \text{ ft})^2] = 3.43 \times 10^{-7} \text{ ft} = 0.105\ \mu\text{m}$

Phase angle:

$\theta_\psi = \text{argument } (A_\psi)$, with a positive and b negative:
$= \arctan[b/a] = \arctan[(3.159 \times 10^{-7} \text{ ft})/(-1.334 \times 10^{-7} \text{ ft})] = -67.1$ deg

CHAPTER 7—DESIGN AND MATERIALS

7.1—Overview of design methods

This section presents general considerations and an overview of three commonly used methods for the design of dynamic equipment foundations: the rule-of-thumb method, the equivalent static loading method, and the dynamic analysis method.

7.1.1 *General considerations*—The objectives of the machine foundation design are to assess the dynamic response of the foundation and verify compliance with the required vibration and structural performance criteria.

Machine foundation design typically includes the following steps:

a) Develop a preliminary size for the foundation using rule-of-thumb approaches (7.1.2.1) or equivalent static loading method (7.1.2.2), past experience, machine manufacturer recommendations, and other available data.

b) Calculate the vibration parameters, such as natural frequency, amplitudes, velocities, and accelerations, for the preliminarily sized foundation (7.1.2.3).

c) Verify that these calculated parameters do not exceed generally accepted limits or project specific vibration performance criteria (7.1.2.3).

d) If necessary, incorporate appropriate modifications in the foundation design to reduce vibration responses to meet the specified vibration performance criteria and cost.

e) Check the structural integrity of the concrete foundation and machine-mounting system.

f) If foundation sizes required are excessively large, determine if vibration mitigation measures such as vibration isolation are required or beneficial.

Preventive measures to reduce vibrations are generally less expensive when incorporated in the original design than remedial measures applied after machinery is in operation. The following are some common means that can be used separately or combined to reduce vibrations:

a) Selection of the most favorable location for the machinery. For example, locating the machine closer to the ground will help reduce rocking vibrations. Another beneficial example is keeping the center of gravity of the machine close to the center of foundation rigidity.

b) Adjustment of machine with respect to speed or balance of moving parts

c) Adjustment of the foundation with respect to mass (larger versus smaller), stiffness, or both. Piles or caissons can increase foundation stiffness substantially.

d) Isolation of the machinery from the foundation using special mountings such as springs and flexible foundation mats

e) Isolation of the foundations from surrounding by physical barriers

7.1.2 *Design methods*—Design methods for the foundations supporting dynamic equipment have gradually evolved over time from an approximate rule-of-thumb procedure to the more technically sound engineering methods. This evolution has been accelerated by the widespread availability of dynamic analysis software. These methods can be identified as follows:

a) Rule-of-thumb

b) Equivalent static loading

c) Dynamic analysis

The selection of an appropriate method depends heavily on machine characteristics, including unbalanced forces, speed, weight, center of gravity location, mounting, importance of the machine, foundation type and size, and required performance criteria.

Design of the foundation starts with the selection and assessment of the foundation type, size, and location. Usually, foundation type is governed by the soil properties and operational requirements. The machine footprint, weight, and unbalanced forces govern the size of the foundation. The location of a foundation is governed by environmental and operational considerations. Thus, the engineer should consider information from the following three categories before the foundation can be preliminarily sized. Note that the following are the sample information required for foundation design.

a) Machine characteristics and machine-foundation performance requirements

- i. Functions of the machine
- ii. Weight of the machine and its moving components
- iii. Location of the center of gravity in both vertical and horizontal dimensions
- iv. Operating speed ranges of the machine
- v. Magnitude and direction of the unbalanced forces and moments
- vi. Limits imposed on the foundation with respect to differential displacement
- vii. Limits with respect to vibration performance criteria: vibration amplitude, velocity, and acceleration

b) Geotechnical information

- i. Allowable soil-bearing capacity
- ii. Allowable pile capacity
- iii. Effect of vibration on the soil—for example, settlement or liquefaction risk
- iv. Classification/type of soil
- v. Modulus of subgrade reaction
- vi. Dynamic soil shear modulus
- vii. Dynamic soil-pile stiffness and damping data (for pile-supported foundations)

c) Environmental conditions

- i. Existing vibration sources such as existing vibrating equipment, vehicular traffic, or construction
- ii. Human susceptibility to vibration or vibration-sensitive equipment
- iii. Flooding
- iv. High water table
- v. Seismic hazard
- vi. Wind
- vii. Snow
- viii. Low or high temperature

Usually, the preliminary foundation size is established using the rule-of-thumb method, and then the performance criteria, for both machine and foundation, are verified using the equivalent static loading method or dynamic analysis. If the equivalent static loading method or dynamic analysis shows that the foundation is inadequate, the engineer revises the foundation size and repeats the analysis.

7.1.2.1 *Rule-of-thumb method*—Rule-of-thumb is the simplest design method for dynamic equipment foundations. The concept of this method is to provide sufficient mass in the foundation block so that the vibration is attenuated and absorbed by the foundation and soil system. Unless otherwise indicated, the rule-of-thumb methods described in the following are referring to block-type or combined block-type foundations supported on soil or piles/caissons. Dynamic equipment supported on elevated structures typically requires dynamic analysis to evaluate its vibration performance adequacy.

For rotating and reciprocating machine foundations, most engineers consider the rule-of-thumb procedure satisfactory for a preliminary foundation sizing. Sometimes engineers use this method to design block-type foundations supporting relatively small machinery, up to 5000 lbm (2270 kg) in mass and having small unbalanced forces. For reciprocating machinery and sensitive machinery, rule-of-thumb procedures by themselves may not be sufficient. A long-established rule-of-thumb for machinery on block-type foundations is to make the mass of the foundation block at least three times the mass of a rotating machine and at least five times the mass of a reciprocating machine. For pile-supported foundations, these ratios are sometimes reduced so that the foundation block mass, including pile cap, is at least 2.5 times the mass of a rotating machine and at least four times the mass of a reciprocating machine. These ratios are machine mass inclusive of moving and stationary parts as compared with the mass of the concrete foundation block. Additionally, many designers require the foundation to be of such weight that the resultant of lateral and vertical loads (static loads) falls within the middle one-third of the foundation base. That is, the net effect of lateral and vertical loads or the eccentricity of the vertical load should not cause lack of compression under any part of the foundation.

The engineer should determine the shape and thickness of the foundation to provide uniform distribution of vertical dead and live loads to the supporting soil or piles, if practical. This can be accomplished by adjusting the length and width of the foundation so the center of gravity of the machine coincides with the center of gravity of the foundation block in plan. A common criterion is that the plan-view eccentricities between the center of gravity of the combined machine-foundation system and the center of resistance (center of stiffness) should be less than 5 percent of the plan dimensions of the foundation. The shape of the foundation should fit the supported equipment requirements. Also, the engineer should provide sufficient area for machine maintenance. The shape of the foundation should adequately accommodate the equipment, including maintenance space if required. The minimum width should be 1.5 times the vertical distance from the machine centerline to the bottom of the foundation block. In any case, the foundation is sized so that the foundation bearing pressure does not exceed the allowable soil-bearing capacity. Depending on machine vendors, allowable soil-bearing pressure for dynamic machines may be limited to one-half of allowable soil bearing pressure for foundations supporting static equipment.

Thickness criteria primarily serve to support a common assumption that the foundation behaves as a rigid body on the supporting material. Clearly, this is a more complex problem than is addressed by simple rules-of-thumb. On soft soils, a thinner section may be sufficient, whereas on stiffer soils, a thicker section might be required to support the rigid body assumption. If the rigid body assumption is not applicable, more elaborate computation techniques, such as finite element methods, should be used. Gazetas (1983) provides some direction in this regard. One rule-of thumb criterion for thickness is that the minimum thickness of the foundation block should be one-fifth of its width (short side), one-tenth of its length (long side), or 2 ft (0.6 m), whichever produces the largest foundation thickness. Another criterion is given in 6.2 as 1/30 of the length plus 2 ft (0.6 m). Normally, dynamically loaded foundations should not be placed on the top of building footings or in such locations that the dynamic effects can transfer into the building footings.

Forging hammer foundations are unique in the sense that design of the foundation is essentially based on the impact energy and characteristics of the forging process. There is no current U.S. document addressing design requirements for forging hammer foundations. Most hammer manufacturers are familiar with DIN 4025 issued in 1958. Although this document has been withdrawn by the German standards organization, it is still widely referenced, as a more current standard is not available. That document is summarized in the following paragraphs for general information.

The required weight of a foundation block resting upon soil should be determined by calculation, and such calculations should consider the effect of vibrations on nearby structures and facilities. One reference equation suggested by DIN 4025 is

$$W_f = 75 \cdot B_r \cdot (v_r/v_o)^2 \qquad (7.1.2.1)$$

Equation (7.1.2.1) assumes an anvil-to-ram weight ratio of 20:1. The minimum foundation weight should be decreased or increased to make up for a lighter or heavier anvil weight relative to the ram weight, respectively.

7.1.2.2 *Equivalent static loading method*—The equivalent static loading method is a simplified and approximate way of applying pseudo-dynamic forces to the machine-supporting structure to check the strength and stability of the foundation. This method is used mainly for the design of foundations for machines weighing 10,000 lbm (4540 kg) or less.

For design of reciprocating machine foundations by the static method, the machine manufacturer should provide the following data:

a) Weight of the machine and baseplate

b) Unbalanced forces and moments of the machine during operation

c) Individual cylinder forces including fluid and inertia effects

For design of rotating machine foundations by the static method, the machine manufacturer should provide:

a) Weight of the machine and base plate

b) Vertical pseudo-dynamic design force

c) Horizontal pseudo-dynamic design forces:

i. lateral force

ii. longitudinal force

Calculated natural frequencies, deformations (displacements, rotations, or both), and forces within the foundation/structure supporting the machine should satisfy established design and performance recommendations outlined in 6.5 and 6.6.

Note that vertical dynamic force is applied normal to the shaft (for a horizontally shafted machine) at the midpoint between the bearings or at the centroid of the rotating mass. If the magnitude of this force is not provided by the machine manufacturer, the engineer may conservatively take it as 50 percent of the machine assembly dead weight. Lateral force is applied normal to the shaft at the midpoint between the bearings or at the centroid of the rotating mass. Longitudinal force is applied along the shaft axis. The engineer can conservatively take both the lateral and the longitudinal force as 25 percent of the dead weight of the machine assembly, unless specifically defined by the machine manufacturer. Vertical, lateral, and longitudinal forces are not considered to act concurrently for many types of rotating machines.

7.1.2.3 *Dynamic analysis method*—Dynamic analysis incorporates more advanced and more accurate methods of determining vibration parameters and, therefore, is often used in the final design stage and for critical machine foundations. Dynamic analysis (or vibration analysis) is almost always required for large machine foundations with significant dynamic forces (6.5 and 6.6).

For the dynamic analysis of reciprocating and rotating machine foundations, the machine manufacturer should at least provide the following data to the extent required by the analysis:

a) Primary unbalanced forces and moments applied at the machine speed over the full range of specified operating speeds

b) Secondary unbalanced forces and moments applied at twice the machine speed over the full range of specified operating speeds

c) Individual cylinder forces, including fluid and inertia effects

Note that a more detailed list can be found in 4.3.

For rotating machinery foundations, the dynamic analysis (6.6) determines the vibration amplitudes based on these dynamic forces. Refer to 4.3 for the dynamic force calculations for both reciprocating and rotating machines. When there is more than one rotor, however, amplitudes are often calculated with the rotor forces assumed to be in-phase and 180 degrees out-of-phase. To obtain the maximum translational and maximum torsional amplitudes, other phase relationships may also be investigated. Alternatively, to obtain maximum values, the loads may be analyzed separately and the peak responses summed.

As discussed in 6.1, a complete dynamic analysis of a system is normally performed in two stages:

1. Frequency analysis: Determination of the natural frequencies and mode shapes of a machine-foundation system

2. Forced vibration analysis: Calculation of the machine-foundation system response caused by the dynamic forces

Determining the natural frequencies and mode shapes of the machine foundation system provides information about the dynamic characteristic of that system. In addition, calculating the natural frequencies identifies the fundamental frequency, usually the lowest value of the natural frequencies. The significance of natural frequencies and mode shapes are discussed in 6.5.

Frequency analysis is also called free vibration analysis. The details of frequency analysis methodology are discussed in 6.5.2 and 6.5.3. Frequency analysis of a soil-foundation-machinery system will provide the dynamic characteristics of the system—for example, natural frequencies, mode shapes, and modal participating mass ratio. The calculated natural frequencies can be used for the resonance check of that system. In addition, the frequencies and modal shapes can be used by the modal superposition/transformation method as the basis to perform forced vibration analysis. In many cases, six frequencies are calculated consistent with six rigid body motions of the overall machine foundation system. Any or all of the six frequencies may be compared with the excitation frequency when checking for resonance conditions.

Forced vibration analysis calculates the machine-foundation system responses caused by the dynamic forces. The results are the specific vibration parameters, such as displacement, velocity, and acceleration of the masses, as well as the internal forces in the members of the machine support system. Then, these vibration parameters are then compared with the defined criteria or recommended allowable values for a specific condition (6.5.1 and 6.6.1), and internal forces are used to check the structural strength of the foundation components. The details of forced vibration analysis methodology are covered in 6.6.2.

Based on results of vibration analyses, the proportions of the foundation (shape, width, length, and depth) foundation embedment depth, and pile arrangement if the foundation is pile-supported, may be modified for reanalysis and reevaluation. This may take numerous iterations until the project-specific vibration acceptance criteria are completely met.

7.2—Concrete

7.2.1 *Concrete performance criteria*—The design of the foundation should withstand all applied loads, both static and dynamic, as defined in Chapter 4. The foundation should act in unison with the equipment and supporting soil or structure to meet the deflection limits specified by the machinery manufacturer or equipment owner. The design service life of a concrete foundation should meet or exceed the anticipated design service life of the equipment installed and resist the cyclic stresses from dynamic loads.

Design of machinery foundations is driven by the unique requirements of the machinery, its size, weight, and the need to control or limit the response to dynamic loading imposed by the equipment. The performance is more dependent on its mass, footprint, and dynamic response as opposed to building-type structures where strength design controls member size and minimum reinforcement requirements imposed by ACI 318. Additionally, safety of the general public is generally not a concern because the foundation is not built to be occupied, as buildings are.

ACI 318 provisions were developed primarily for commercial and office building structures. For these structures, member sizes are selected based on the minimum design strengths to resist environmental and gravity loads. Reinforcement is sized to resist temperature and shrinkage, compression, tension, shear, and torsion. Member sizes may be larger than required by strength to satisfy deflection and drift. High occupancy is expected and minimum reinforcement using ACI 318 criteria is maintained to provide strength for service loads and ductility during seismic events to provide a certain degree of over-strength for increased design service life safety capabilities of the structure.

There is limited guidance currently available on how to select minimum reinforcement in the design of oversize sections for machinery foundations. In the absence of this information, users often default to ACI 318. Applying ACI 318 provisions to oversize members, however, can result in very conservative and costly designs.

Some equipment vendors may require stresses calculated under service loads to meet their specific allowable stress acceptance criteria. For some equipment foundations, there is neither a published standard nor a clear industry consensus as to which type of performance criteria is appropriate. In those cases, foundation designer, equipment vendor, and owner should seek a clear understanding and mutual agreement of the foundation performance criteria.

Concrete cracking, regardless of the causes of the cracking, can have negative effects on foundation durability. Unlike statically loaded foundations, cracking of concrete in foundations supporting dynamic equipment can also cause a significant reduction in foundation stiffness and affect vibration performance. Some equipment vendors require foundation design to meet their specific cracking acceptance criteria. Minimizing shrinkage and temperature cracking is critical to avoid stiffness loss and concrete deterioration.

In foundations thicker than 4 ft (1.2 m), the engineer may use the minimum reinforcing steel suggested in ACI 207.2R. The concrete used for the dynamic equipment foundations should also meet the material requirements of ACI 351.2R.

7.2.2 *Concrete strength*—The basic principles of reinforced concrete design are used in the design of foundations supporting dynamic equipment. Engineers working with specific equipment may apply additional criteria based on their experience. Such equipment includes forging hammers, turbine generator systems, and large compressors.

The general equipment foundation includes some components that primarily act in flexure, others that are primarily axial, and others may act primarily in shear. Applicable sections of ACI 318 are often used to establish minimum requirements for axial, flexural, and shear reinforcement. In some cases, engineering firms have supplemented these criteria with internal criteria or internal interpretations of the ACI 318 requirements for foundation structures that are nonbuilding structures. For thick sections such as turbine-generator pedestals or foundation mats over 4 ft (1.2 m) thick, special criteria involving location of reinforcement or minimum reinforcement may be identified more in line with mass concrete construction. Such criteria are typically structure-specific and, thus, are not extendable to the broad class of foundations addressed in this document.

Largely because of the broad range represented in this class of construction, accepted standards have not evolved. For example, there is no specific minimum amount of reinforcement applicable to all the various equipment foundation designs. Most of these foundations are designed for mass and stiffness and not necessarily for strength and, thus, reinforcement is specified by owner or vendor requirements, historical practices, or rules-of-thumb. A minimum concrete strength of 4000 psi (28 MPa) is commonly specified for dynamic equipment foundations and foundations supporting industrial equipment, unless a higher strength is required for durability.

7.2.2.1 *Consideration of durability*—In most cases, the quality and durability of the concrete is considered more critical to good performance than strength. Concrete durability criteria are, however, becoming more stringent. Even if concrete has adequate strength to resist design loads, concrete strength higher than 4000 psi (28 MPa) may be required for applications where the concrete foundation is subjected to more severe weather (freezing and thawing) exposure or sulfate exposure. According to ACI 318, the minimum concrete strength is 4500 psi (31 MPa) for the concrete exposed to Exposure Classes F2, S2, and S3, and 5000 psi (35 MPa) for concrete exposed to Exposure Classes C2 and F3.

7.2.2.2 *Consideration of thermal effects*—Some types of dynamic equipment also develop greater-than-normal thermal conditions, with concrete surface temperatures exceeding 150°F (66°C) around and within the foundation. This is especially true for combustion turbines, steam turbines, and compressors. The engineer should address the effects of these thermal conditions in the design phase. Inadequate consideration of the thermal effects can lead to early cracking of the foundation, which is then further increased by the dynamic effects.

Calculation of thermally-induced bending requires proper determination of the heat distribution through the thickness of the foundation. Heat transfer calculations can also be performed either one-, two-, or three-dimensional thermal analysis. ACI 307 and ACI 349.1R provide some guidance that can be extrapolated to hot equipment. Effective methods of controlling the thermal effects include:

a) Providing sufficient insulation between the hot equipment and the concrete

b) Providing sufficient cover to the reinforcement so that thermally-induced cracking neither degrades the bond of the reinforcement nor increases the exposure of the reinforcement to corrosives

c) Providing sufficient reinforcement to control the growth of thermal-induced cracks

d) Providing high-temperature-resistant concrete

7.2.3 *Material dynamic properties*

7.2.3.1 *Dynamic modulus of elasticity*—Established relationships suggest that the ratio of dynamic to static modulus can vary from 1.1 to 1.6, with significant variation with age and strength. In practice, engineers treat this strain-rate effect differently. In some firms, engineers perform calculations using the higher dynamic modulus whereas other firms may consider the difference unimportant and use the static modulus from ACI 318. Note that the static modulus of ACI 318 is defined as the slope of stress-strain curve drawn from a stress of zero to a compressive stress of $0.45f_c'$. This modulus of elasticity is also referred to as static secant modulus. The distinction is more important for elevated tabletop-type foundations where the frame action of the structure is stiffer if a dynamic modulus is used. The difference in using static and dynamic concrete modulus of elasticity can also be important in compressor foundations, where the stiffness of the machine frame should be evaluated against the stiffness of the concrete structure. For simple, block-type foundations, the concrete modulus of elasticity has no real effect on the design.

For large steam turbine generator foundations, however, ASCE Task Committee (1987) does not recommend the use of a higher dynamic modulus of elasticity because of the considerations of concrete cracks that may often be present in these heavy massive structures.

Many tests were carried out by researchers to correlate static (E) and dynamic (E_d) moduli for concrete. One commonly used formula for the dynamic modulus of elasticity of normalweight concrete is shown in the following equation, which yields a dynamic modulus equal to 1.38 times the static modulus of ACI 318.

$$E_d = 79{,}000(f_c')^{0.5} \quad \text{(psi)}$$
$$E_d = 6550(f_c')^{0.5} \quad \text{(MPa)} \qquad (7.2.3.1)$$

Dynamic concrete modulus may also be determined by other appropriate means (Neville 1997; BS 8110-2; USACE EM 1110-2-6051; Popovics 2008).

Dynamic modulus of elasticity is applicable to only response spectrum and time-history dynamic analysis load cases (as they have rapid rate of changes in the loads). There is no common agreement to use it in the static equivalent analysis, steady-state dynamic analysis, or both, unless required by machine manufacturers.

In the load combinations composed of static and dynamic load cases, the engineer may need to investigate and develop design values by two separate analyses (static with static modulus/properties and dynamic with dynamic modules/properties of material).

7.2.3.2 *Concrete material damping*—Concrete material damping typically does not have much impact on the vibration performance of block-type foundations. Concrete material damping is very small compared to soil damping (geometric damping plus soil material damping). In that case, the material damping of concrete may be neglected. In contrast, concrete material damping can play an important role in vibration performance of elevated tabletop-type foundations. For foundation vibration analysis, concrete material damping ratio is typically taken as 2 percent due to low stress levels in the foundations. ASCE Task Committee (1987) recommends that damping of concrete structural members/elements should be taken as no greater than 2 percent. Some engineers conservatively use 1 percent of concrete material damping for the dynamic equipment that is highly vibration-sensitive.

7.2.3.3 *Concrete Poisson's ratio*—For an isotropic and linear-elastic material, Poisson's ratio is constant; however, in concrete, Poisson's ratio may be influenced by specific conditions. For stresses for which the relationship between the applied stress and the longitudinal strain is linear, however, the value of Poisson's ratio for concrete is approximately constant. Depending on the properties of the aggregate used, Poisson's ratio of concrete lies generally in the range of 0.15 to 0.22 when determined from strain measurements under a compressive load. The value of Poisson's ratio under tensile load appears to be the same as in compression. Existing research (Popovics 2008) reports that the dynamic and static values of Poisson's ratio are not strongly related to each other and dynamic Poisson's ratio can be as high as 0.25. However, foundation designers commonly use 0.15 to 0.2 as concrete Poisson's ratio for foundation dynamic analysis.

7.2.3.4 *Moment of inertia of members*—Gross moment of inertia (I_g) calculated based on an uncracked concrete section is typically used for modeling of block-type foundations and tabletop foundations because of massive member size and low stresses. If cracked members are to be considered, however, reduced moment of inertia as determined in accordance with 6.6.3 of ACI 318-14 is typically used.

7.3—Reinforcement

7.3.1 *Reinforcing steels*—As with most construction, ASTM A615/A615M Grade 60 reinforcing steel is most commonly used for dynamic machine foundations. Good design practice requires particular attention to the detailing of the reinforcement, including proper development of the bars well into the block of the concrete, avoidance of bar ends in high-stress regions, and appropriate cover.

Where concrete is exposed to severe weather, sulfate exposure, or both, reinforcing steel may require corrosion protection. Depending on the corrosion severity, protection measures could include use of epoxy-coated, metallic-coated, and solid metallic reinforcing bars (such as galvanized steel bars and stainless steel bars); use of FRP bars; use of corrosion inhibitors (such as calcium nitrates or other organic corrosion inhibitors); use of concrete membranes or concrete sealers; or providing catholic protection.

7.3.2 *Minimum reinforcing requirements*—Industry practice on the minimum reinforcement used varies widely, depending on type of dynamic equipment foundations, type of members, and the preference of firms.

In foundations thicker than 4 ft (1.2 m), the engineer commonly uses the minimum reinforcement suggested in ACI 207.2R for mass concrete. Section 7.12.2 of ACI 350-06 provides more recent alternative criterion, which states that concrete sections that are at least 24 in. (600 mm) thick may have the minimum shrinkage and temperature reinforcement based on a 12 in. (300 mm) concrete layer at each face.

For large turbine generator foundations, some firms specify a minimum reinforcement of 3.1 lbf/ft^3 (50 kgf/m^3 or 0.64 percent by volume) for pedestals and 1.9 lbf/ft^3 (30 kgf/m^3 or 0.38 percent by volume) for foundation slabs. Others prefer to use a larger minimum reinforcement of 3.1 lbf/ft^3 (50 kgf/m^3) in the foundation slabs. Some European equipment vendors require the use of multiple layers of reinforcing steel spaced in all three directions. For compressor blocks, some firms suggest 1 percent reinforcement by volume consisting of multi-layers of reinforcement instead of reinforcement only at faces of concrete and may post-tension the block. Many engineers recommend additional reinforcement, such as hairpin bars, around embedded anchor bolts to ensure long-term performance. The criteria and presentations in ACI 351.2R for static equipment also can be applied to dynamic equipment foundations.

Design of forging hammer foundations considers a statically equivalent load determined from the impact energy and characteristics of the forging process in addition to other design basis loads. Minimum reinforcement of the foundation block is set at 1.56 lbf/ft^3 (25 kgf/m^3) or 0.32 percent by volume. This reinforcement should be distributed in all three directions throughout the foundation block. The upper layer of steel should be capable of resisting 1 percent of the statically equivalent load applied in any horizontal direction. Bending and shear effects should be addressed in the layout and design of the reinforcement. In large hammer foundations, reinforcement is often installed in all three orthogonal directions and diagonally in the horizontal and vertical planes. Suitable development of the reinforcement is very important. Excessive reinforcement can create constructability and quality problems and should be avoided.

7.3.3 *Common reinforcing steel design practice for concrete pedestals*—ACI 351.2R provides a summary of commonly used reinforcing steel design practice for concrete pedestals supporting static equipment. These practices are equally applicable to foundations supporting dynamic equipment.

7.4—Machine anchorage

7.4.1 *Performance criteria*—The major components of machine anchorage are the anchor bolts and the support system directly under the machine frame at the anchor bolt location. Selection of the support system is generally determined by the equipment supplier considering the support requirements for the equipment base along with any requirement to ventilate the bottom of the equipment. Support systems range from a full bed of grout to various designs of soleplates and chocks, fixed or adjustable. Figure 7.4.1 shows various types of machine frame support system using epoxy grout, chocks, and shims.

One of the important performance criteria on seismic design, as required in ASCE/SEI 7, specifies that the component attachments shall be bolted, welded, or otherwise positively fastened without consideration of frictional resistance produced by the effects of gravity.

The structural performance criteria for anchor bolts holding dynamic machinery require that sufficient clamping force be available to maintain the critical alignment of the machine. The clamping force should allow smooth transmission of unbalanced machine forces into the foundation so that the machine and foundation can act as an integrated structure. Generally, higher clamping forces are preferred because they result in less vibration being reflected back into the machine. In the presence of unbalanced forces, a machine that has a low clamping force expressed in terms of bearing pressure of 400 psi (2.8 MPa) at the machine support points can vibrate more than the same machine with high clamping forces with a bearing pressure of 1000 psi (7 MPa). Precision machines in the machine tool industry sometimes have clamping forces that result in bearing pressure as high as 2000 psi (14 MPa) to minimize tool chatter. Instead of a more refined method, designing for clamping force that is 150 percent of the anticipated normal operating bolt force is good practice. A minimum anchor bolt clamping force of 15 percent of the bolt material yield strength is often used if specific values are not provided by the equipment manufacturer. Higher values are appropriate for more aggressive machines. Clamping force, also known as preload or pretensioning, is developed by tensioning the anchor bolt.

7.4.2 *Anchorage materials and styles*—There are various styles (types) of anchor bolts, including J style, L style, P style, N style, PH style, PN style, S style, and post-installed anchors that may be used for anchorage of equipment support. A detailed discussion of anchorage materials, anchorage styles, and their design can be found in 5.2 of ACI 351.2R-12.

7.4.3 *Anchor bolt tensioning*—To avoid slippage under dynamic loads at any interface between the frame and chock and soleplate, or chock and foundation top surface, the normal force at the interface multiplied by the effective coefficient of friction should exceed the maximum horizontal dynamic force applied by the frame at the location of the tie-down.

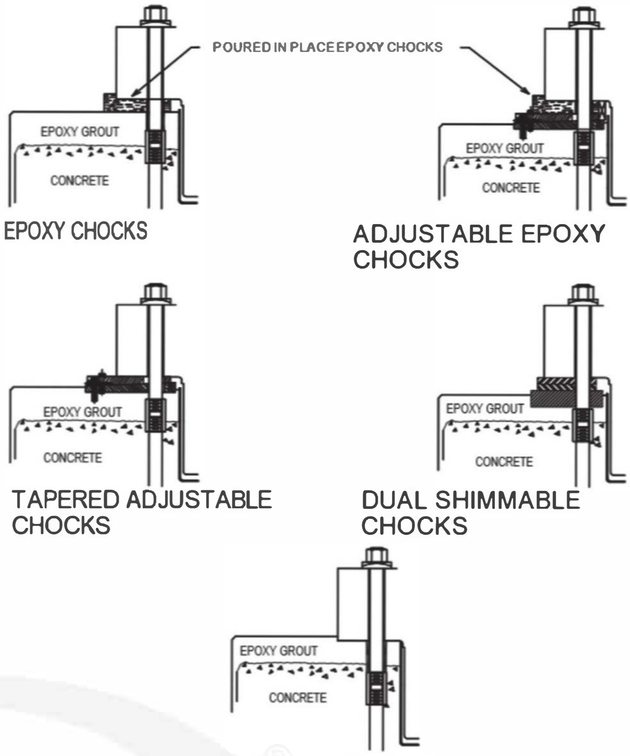


Fig. 7.4.1—Types of machinery frame support systems (courtesy of Robert L. Rowan & Associates).

7.4.3.1 *Determination of minimum anchor bolt tension force*—In general:

$$F_r = \mu(T_{min} + W_a) \text{ or } T_{min} = F_r/\mu - W_a \qquad (7.4.3.1)$$

An anchor bolt and concrete anchorage system that has long-term tensile strength in excess of T_{min} and maintains preload at or above this tension, coupled with a chock interface whose coefficient of friction equals or exceeds μ, will withstand the force F_r. A conservative approach neglects W_a (assumes it to be zero) because distortion of the frame or block may reduce the effective force due to weight at any one anchorage location.

In the case of machines, such as reciprocating gas compressors, gas loads or inertia loads may dictate the required frictional holding capacity F_r (Smalley and Pantermuehl 1997), depending on the location of the anchor bolt. Because holes in the frame and crosshead guide establish bolt diameter, the bolt strength determines the maximum possible preload.

The Gas Machinery Research Council (GMRC) research program (Smalley 1997) has developed data for friction between common chock interface materials, including steel/cast iron, steel/steel, epoxy/cast iron, epoxy/steel, and epoxy/epoxy in both dry and when-oil-is-present conditions, using sizes that come close to those typical of compressor mounting practice. This report presents some values for breakaway friction coefficients, including a range from 0.22 to 0.41 for dry interfaces between cast iron and various epoxy products and a value of 0.19 for cast iron on cold-rolled steel. The presence of oil in the sliding interfaces reduces the friction coefficient for cast iron on epoxy to a range from 0.09 to 0.15 and to a value of 0.14 for cast iron on cold-rolled steel. Thus, maintaining an oil-free interface greatly enhances frictional holding capacity.

7.4.3.2 *Preload loss and monitoring preloads*—Anchor bolts always lose a portion of the preload, both in the first 24 hours after tightening and then during operation. Usually, at least one retightening is required until the preload stabilizes. For dynamic equipment, vibrations tend to promote a loss of tension in anchor bolts larger than static equipment anchorage. Loss of preload can increase vibration levels and adversely affect machine vibration performance. Some manufacturers recommend periodic retightening of anchor bolts to the original preload level for their vibration-sensitive equipment.

7.4.4 *Anchor bolt depth/length/style*—For dynamic equipment support, a good common practice is to make the anchor bolt as long as possible so the anchoring forces are distributed lower in the foundation or ideally into the concrete mat foundation below the foundation pedestal. Alternatively, anchor bolts may stay within the foundation pedestal, but additional dowels are provided around anchor bolts to help transfer tension into the foundation mat. Anchor bolts that are designed to exactly match a ductile failure criterion and just long enough so that the concrete pullout strength equals the strength of the steel may be still too shallow for dynamic machinery foundations. Cracking of the upper concrete has been a common problem when shallow embedment depths are used.

There are additional benefits to using a long anchor bolt. Such systems exhibit greater tolerance to grout creep, that is, less loss of preload from creep. In addition, the lower termination point, in the foundation or in the mat foundation below, moves the potential site for any crack initiation away from the dynamic loads imposed by the machine and from sources of oil.

In addition to the depth, the engineer should pay attention to the bolt style. J- and L-style bolts can straighten out and pull out of concrete foundations before their maximum tensile capacity is reached. Many engineers prohibit their use with dynamic machinery. Expansion shell anchors should only be used where they have been tested by the manufacturer and approved for the vibratory condition of the specific dynamic machinery application.

It is worth pointing out that another good practice is the use of sleeves to allow elongation of the anchor bolt over a longer development length, especially with epoxy grouts, to increase clamping force without causing undue localized stress on the anchor bolt. The capacity of anchor bolts should be determined as described in ACI 351.2R and in accordance with ACI 318.

7.5—Elastic support systems

For the application and design of elastic support systems, refer to 3.3.3, 3.3.5, and 6.3.3.

7.6—Grout

There are two basic types of grout: cementitious (cement-based) grouts and epoxy grouts. Both types of grout have limitations that should be considered when specifying a grout for a specific project.

ACI 351.1R provides a detailed discussion of the properties of both epoxy and cementitious grouts. Several reports (Smalley and Pantermuehl 1997; Smalley 1997; Pantermuehl and Smalley 1997a,b) address friction properties, creep properties, and methods of engineering and assessing epoxy grouts and chock material for reciprocating compressor applications. ACI 351.1R also provides detailed recommendations on grout material requirements, design, details, and placement of grouts. Additionally, ACI 351.4 and ACI 351.5 provide specifications for installation of cementitious grouts and epoxy grouts, respectively. Typically, the foundation designer is responsible for the selection of the grout for vibratory equipment foundation unless it has been specified in project specifications. Considerations that are important in selection of grouts for dynamic equipment foundations are:

a) The type of load that will be applied to the base plate
b) The operating temperature at which the grout must perform
c) Workability
d) Service history
e) Recommendations of equipment manufacturer

The grout chosen should provide the long-term strength to carry the applied load from the machine mounts. For dynamic equipment foundations, engineers can specify either cementitious grout (also called hydraulic machine base grout) or epoxy grout. However, for dynamic machines that produce large impact loads and shock loading, such as produced by reciprocating machines or hammers/crushers, epoxy grouts are often specified because of better resistance to vibration and impact loads and better chemical resistance to process fluids and lubricating oils.

Generally speaking, epoxy grouts can perform at temperatures below 130°F (55°C), whereas cementitious grout can operate at temperatures up to 400°F (200°C). At service temperatures of 150°F (66°C) or higher, epoxy grouts can experience significant creep, thus increasing static deformation at supports and limiting the usage of epoxy grouts to where operating temperature is not severe. However, epoxy grouts are less rigid than cementitious grouts and have higher resistance to chemicals, shock, and vibratory loads.

Cementitious grouts can have compressive strength as high as 8000 psi (56 MPa), but are low in tensile and flexural strength, thereby limiting their use for dynamic machines to smooth running rotating machines such as fans, motors, electric generators, and turbine generators. Cementitious grouts are generally preferred to epoxy grouts for supporting static equipment and for smooth running dynamic equipment because they are easier to place, cost less, and creep less under temperature when compared to epoxy grouts. This design practice is followed by many turbine generator manufacturers, fan manufacturers, motor manufacturers, and engineers responsible for foundation design where flowable nonshrink cementitious grouts are commonly specified.

7.7—Seismic design considerations

Seismic design of dynamic equipment foundations should follow the seismic design requirements stipulated in ASCE/SEI 7-16 Chapter 13 as components or Chapter 15 as nonbuilding structure components. A detailed discussion about seismic loads can be found in 4.4.2.

Large tabletop-frame-type foundations such as steam turbine generator pedestal foundations can be treated as building-like nonbuilding structures using 15.5 of ASCE/SEI 7-16 for their seismic design.

Due to the massive member sizes of these frame-type tabletop foundations where member sizes are typically governed by stiffness/mass rather than strength, providing ductile detailing can be difficult. Industry experience in engineering and detailing of elevated tabletop-type foundations for high seismic regions is limited at this time. Limited experience thus far seems to indicate that it is more cost-advantageous to design a less ductile frame system that results in higher design seismic loads.

7.8—Fatigue considerations

For many dynamic equipment foundations, the cyclic stresses are small, and engineers choose not to perform any specific fatigue stress calculations. Other equipment can require more significant consideration of cyclic stresses. One example is the foundation of a wind turbine, which is required to withstand numerous cycles of small to moderate stress range and hundreds of cycles of large stress range. In such cases, ACI 215R and other fatigue codes provide guidance, particularly where the flexural characteristics of the foundation are most important. Note that ACI 215R first published in 1974 was revised in 1992 and has not been updated since. Engineers may refer to European codes for newer fatigue provisions. EN 1992-1-1 and DNV-OS-C502 seem to provide the most comprehensive fatigue design guidelines for concrete structures.

Aside from performing detailed fatigue stress evaluation, some firms choose to employ simplified methods to implicitly or explicitly address fatigue. These include:

a) Proportioning sections to resist all conventional loads plus three times the dynamic load

b) Designing such that concrete modulus of rupture is not exceeded while including the inertial loads from the concrete motion. In certain cases, the calculated modulus of rupture is reduced by 50 percent to approximate permissible stresses reduced for fatigue

c) Reducing by as much as 80 percent the strength reduction factors specified by ACI 318

d) Recognizing that cracking is less likely in structures built with clean, straight lines and not having reentrant corners and notches.

7.9—Special considerations for compressor block post-tensioning

Some engineers prefer that block-type compressor foundations be post-tensioned to provide residual compressive stress that will prevent the generation of cracks. Shrinkage cracks or surface drying cracks are expected in any concrete block-type foundation, especially when the water content in the concrete mixture is excessive. With the addition of subsequent vibration, these cracks propagate, allowing oil to penetrate the block and eventually destroy the integrity of the foundation. Post-tensioning puts the block in compression, offsetting the dynamic and shrinkage stresses. When horizontal post-tensioning rods are placed one-third the distance from the top of the block, a triangular compression stress distribution can be idealized. This idealization maximizes the compression at the top where it is needed the most. An average horizontal pressure of 100 psi (690 kPa) at the level of the prestress translates to a horizontal pressure of 200 psi (1380 kPa) at the top, providing the necessary residual compression.

Vertical post-tensioning rods are anchored as deep as possible into the mat foundation and are sleeved or taped along their length to allow them to stretch during post-tensioning. The embedded end is anchored by a nut with a diameter twice the rod diameter and a thickness 1.5 times the rod diameter. Horizontal rods are not bonded (sleeved) and are anchored at each end of the block through thick bearing plates designed to distribute the load to the concrete. High-strength steel rods are recommended for post-tensioning compressor blocks (Smalley and Pantermuehl 1997).

The concrete should have a 28-day strength of at least 3500 psi (24 MPa) and be reasonably free of shrinkage cracks when cured. Compliance with the ACI 318 provisions is commonly mandated for these systems.

7.10—Sample calculations

A total of five examples are presented in the following sections. Examples 1 to 3 are to illustrate application of the three design methods as described in 7.1.2. Example 4 is provided to show determination of anchor bolt tension forces. It should be pointed out that Examples 3a and 3b represent a typical simplified dynamic analysis with rigid foundation assumption. The method is well suited for block-type foundations. Where the foundation does not meet rigid foundation assumption, however, a more refined dynamic analysis (finite element analysis) will be needed.

The design input of the first three examples is from an indoor block-type foundation supporting a centrifugal dynamic machine. Wind load and seismic load do not govern the design, as the plant is located in a low seismic region.

Note that SI values are not provided in sample calculations, and U.S. customary units only are used to simplify the calculations.

Given conditions:

Allowable soil bearing capacity: $P_{all} = 5$ ksf

Soil modulus of subgrade reaction: $k_s = 36$ lbf/in.^{3}

Allowable foundation settlement: $S_{all} = 1$ in.

Machine speed: $f_o = 600$ rpm

Machine weight: $W_m = 98{,}000$ lbf

Machine unbalance force at the center of gravity of the machine: $F_o = 2000$ lbf

Machine footprint: length: $L_{mf} = 30$ ft, width: $B_{mf} = 16$ ft

Distance of machine shaft centerline to top of foundation: d_{mf} = 10 ft

Concrete 28-day strength: f_c' = 4000 psi

Concrete unit weight: γ_c = 150 lbf/ft^3

Dynamic acceptance criteria:

Mass ratio of foundation to machine: 3

Frequency separation: Fundamental frequencies of foundation should be 25 percent away from the machine operating speed of 600 rpm.

Vibration limit: Peak-to-peak (p-p) displacements at the center of gravity of the machine should not exceed 1.5 mils at machine operating speed of 600 rpm under a machine unbalance force of 2000 lbf.

7.10.1 *Example 1: design by the rule-of-thumb method*

Initial length of foundation mat:

L_M = 1 ft + L_{mf} + 1 ft = 1 ft + 30 ft + 1 ft = 32 ft (for typical clearance)

Initial width of foundation mat:

B_{M1} = 1 ft + B_{mf} + 1 ft = 1 ft + 16 ft + 1 ft = 18 ft (for typical clearance)

Select initial mat foundation thickness:

According to the four minimum thickness proportioning rule-of-thumb rules:

$T_{M1} = L_M/10$ = 32 ft/10 = 3.2 ft

$T_{M2} = B_{M1}/5$ = 18 ft/5 = 3.6 ft

T_{M3} = 2 ft

$T_{M4} = B_{M1}/30$ + 2 ft = 18 ft/30 + 2 ft = 2.6 ft

Select T_{M2} = 3.6 ft, which is equal or greater than all the aforementioned values.

Check the initial width adequacy using the following rule-of-thumb rule:

B_{M_req} = 1.5 × (d_{mf} + T_{M2}) = 1.5 × (10 ft + 3.6 ft) = 20.4 ft > B_{M1}

As the minimum foundation width of 20.4 ft exceeds the initial width of 18 ft, and considering the site space restriction, revise foundation width to B_M = 20 ft.

Recheck mat foundation thickness due to larger width:

$T_{M\_req} = B_M/5$ = 20 ft/5 = 4 ft; revise foundation thickness to T_M = 4 ft.

Check foundation mass (or weight) adequacy:

Weight of concrete mat foundation:

$W_f = (L_M \times B_M \times T_M) \times \gamma_c$ = (32 ft × 20 ft × 4 ft) × 150 lbf/ft^3 = 384,000 lbf

Mass (or weight) ratio of concrete foundation to machine:

$MR = W_f/W_m$ = 384,000 lbf/98,000 lbf = 3.9 > 3

Thus, the foundation with size selected meets the minimum mass ratio requirement.

This example demonstrates that the rule-of-thumb method is simple and can be easily applied to establish the initial foundation size. The initial foundation sizes can be verified using the equivalent static loading method described in 7.1.2.2 and Example 2 (7.10.2).

7.10.2 *Example 2: equivalent static loading method*

Vertical pseudo-dynamic design force:

$F_{pv} = 0.5 \times W_m$ = 0.5 × 98,000 lbf = 49,000 lbf

Lateral and longitudinal pseudo-dynamic design forces:

$F_{pl} = 0.25 \times W_m$ = 0.25 × 98,000 lbf = 24,500 lbf

Overturning moment:

$M_o = F_{pl} \times (d_{mf} + T_M)$ = 24,500 lbf × (10 ft + 4 ft) = 343,000 lbf-ft

Resisting moment:

$M_{res} = (W_m + W_f) \times B_M/2$ = (98,000 lbf + 384,000 lbf) × 20 ft/2 = 4,820,000 lbf-ft

Overturning check:

$M_{res} > M_o$ **OK**

Maximum bearing pressure:

$P_{max} = (W_f + W_m + F_{pv})/(L_M \times B_M)$

= (384,000 lbf + 98,000 lbf + 49,000 lbf)/(32 ft× 20 ft)

= 0.83 ksf < 0.5 × P_{all} = 2.5 ksf

Note that the maximum soil bearing pressure due to normal operating is 0.83 ksf, which is less than half of the allowable soil bearing capacity of 5 ksf.

Static soil stiffness:

$K_s = k_s \times (L_M \times B_M)$

= 36 lbf/in.^{3}× (32 × 12 in. × 20 × 12 in.) = 3,318,000 lbf/in.

Maximum foundation settlement:

$S_{max} = (W_f + W_m + F_{pv})/K_s$

= (384,000 lbf + 98,000 lbf + 49,000 lbf)/(3,318,000 lbf/in.)

= 0.16 in. < S_{all} (= 1 in.)

Note that the equivalent static loading method can verify the foundation configuration design based on the soil bearing, settlements, and foundation stability, but the dynamic analysis method (Example 3a and 3b [7.10.3 and 7.10.4]) will be needed for evaluation of the foundation dynamic performance acceptance.

7.10.3 *Example 3a: dynamic analysis method-frequency check*

Frequency acceptance criteria:

Fundamental frequencies ω_u of the foundation system should be 25 percent away from the machine speed of 600 rpm (10 Hz).

ω_u < 0.75 × 600 rpm = 7.5 Hz

or

ω_u > 1.25×600 rpm = 12.5 Hz

The soil spring stiffness was calculated at the center of resistance under 600 rpm using commercial software:

Soil vertical stiffness K_V = 2.629 × 10^8 lbf/ft

Soil horizontal stiffness K_H = 2.467 × 10^8 lbf/ft

Soil rocking stiffness K_R = 5.292 × 10^{10} lbf-ft/rad

Soil cross stiffness K_C = -4.933 × 10^8 lbf/rad

Total mass of the foundation system:

$m = (W_f + W_m)/g = (384{,}000\ \text{lbf} + 98{,}000\ \text{lbf})/32.17\ \text{ft/s}^2 = 14{,}982.9\ \text{lbf-s}^2/\text{ft}$

Calculate the undamped vertical frequency using Eq. (6.4.1f):
$\omega_v = \text{sqrt}[K_V/M] = \text{sqrt}[(2.629{\times}10^8\ \text{lbf/ft})/(14{,}982.9\ \text{lbf-s}^2/\text{ft})] = 132.5\ \text{rad/s} = 21.1\ \text{Hz}$

Calculate the undamped lateral frequency using Eq. (6.4.1f):
$\omega_h = \text{sqrt}[K_H/\text{M}] = \text{sqrt}[(2.467 \times 10^8\ \text{lbf/ft})/(14{,}982.9\ \text{lbf-s}^2/\text{ft})] = 128.3\ \text{rad/s} = 20.4\ \text{Hz}$

Mass moment of inertia about the rocking axis at center of gravity:
$I_R = W_f \times [(B_M^2 + T_M^2)/12 + (h - T_M/2)^2]/g + W_m \times (d_{mf} + T_M - h)^2/g$
$= 384{,}000\ \text{lbf} \times \{[(20\ \text{ft})^2 + (4\ \text{ft})^2]/12 + (4.44\ \text{ft} - 4\ \text{ft})^2\}/32.17\ \text{ft/s}^2 + 98{,}000\ \text{lbf} \times (10\ \text{ft} + 4\ \text{ft} - 4.44\ \text{ft})^2/32.17\ \text{ft/s}^2$
$= 763{,}281\ \text{lbf-s}^2\text{-ft}$

Calculate the undamped rocking frequency using Eq. (6.4.1f):
$\omega_\phi = \text{sqrt}[K_R/I_R] = \text{sqrt}[(5.292 \times 10^{10}\ \text{lbf/ft})/(763{,}281\ \text{lbf-s}^2/\text{ft})] = 263.3\ \text{rad/s} = 41.9\ \text{Hz}$

Calculate the coupled horizontal translation and rocking frequency as described in 6.4.1:
Distance of base to center of gravity of foundation system:
$h = 4.44$ ft

Calculate stiffness at center of gravity of foundation system:
$K_H' = K_H = 2.467 \times 10^8$ lbf/ft
$K_R' = K_R + K_H \times h^2 - 2 \times K_C \times h$
$= 5.292 \times 10^{10}\ \text{lbf-ft/rad} + 2.467 \times 10^8\ \text{lbf/ft} \times (4.44\ \text{ft})^2 - 2 \times (-4.933 \times 10^8\ \text{lbf/rad}) \times 4.44\ \text{ft}$
$= 6.22 \times 10^{10}$ lbf-ft/rad
$K'_C = K_C - K_H \times h = -4.933 \times 10^8\ \text{lbf/rad} - 2.467 \times 10^8\ \text{lbf/ft} \times 4.44\ \text{ft}$
$= -1.59 \times 10^9$ lbf-ft/rad

Quadratic solution of Eq. (6.5b):
$B_1 = -K'_H/M - K'_R/I_R = -(2.467{\times}10^8\ \text{lbf/ft})/(14982.9\ \text{lbf-s}^2/\text{ft}) - (6.22{\times}10^{10}\ \text{lbf-ft/rad})/(763{,}281\ \text{lbf-s}^2\text{-ft}) = -9.79{\times}10^4\ \text{s}^{-2}$
$B_2 = (K'_H \times K'_R - K'_C{}^2)/(M \times I_R) = [2.467 \times 10^8\ \text{lbf/ft} \times 6.22 \times 10^{10}\ \text{lbf-ft/rad}) - (-1.59{\times}10^9\ \text{lbf-ft/rad})^2]/(14{,}982.9\ \text{lbf-s}^2/\text{ft} \times 763{,}281\ \text{lbf-s}^2\text{-ft}) = 1.12 \times 10^9\ \text{s}^{-4}$
$\omega_1^2 = [-B_1 + (B_1^2 - 4 \times B_2)^{1/2}]/2 = \{-(-9.79 \times 10^4\ \text{s}^{-2}) + [(-9.79 \times 10^4\ \text{s}^{-2})^2 - 4 \times 1.12 \times 10^9\ \text{s}^{-4}]^{1/2}\}/2 = 84{,}678.67\ \text{s}^{-2}$
$\omega_1 = 291\ \text{rad/s} = 46.3\ \text{Hz}$
$\omega_2^2 = [-B_1 - (B_1^2 - 4 \times B_2)^{1/2}]/2 = \{-(-9.79{\times}10^4\ \text{s}^{-2}) + [(-9.79{\times}10^4\ \text{s}^{-2})^2 - 4 \times 1.12 \times 10^9\ \text{s}^{-4}]^{1/2}\}/2 = 13{,}231.78\ \text{s}^{-2}$
$\omega_2 = 115\ \text{rad/s} = 18.3\ \text{Hz}$

The lowest fundamental frequency is the coupled horizontal translation and rocking frequency (18.3 Hz), which is sufficiently higher than 12.5 Hz (125 percent of the machine speed). Therefore, this foundation design meets the dynamic frequency acceptance criteria.

7.10.4 *Example 3b: Dynamic analysis method—calculation of vibration magnitudes*—All design input are the same as in Examples 1, 2, and 3a except that the foundation should meet the following allowable vibration magnitude criterion:

Vibration displacements, p-p, at the center of gravity of the machine ≤ 1.5 mils at machine operating speed of 600 rpm under a dynamic machine unbalance force of 2000 lbf.

The soil spring stiffness was calculated at the center of resistance under 600 rpm using software:

Soil vertical stiffness: $K_V = 2.629 \times 10^8$ lbf/ft
Soil horizontal stiffness: $K_H = 2.467 \times 10^8$ lbf/ft
Soil rocking stiffness: $K_R = 5.292 \times 10^{10}$ lbf-ft/rad
Soil cross stiffness: $K_C = -4.933 \times 10^8$ lbf/rad
Soil vertical damping: $C_V = 3.9 \times 10^6$ lbf-s/ft
Soil horizontal damping: $C_H = 2.522 \times 10^6$ lbf-s/ft
Soil rocking damping: $C_R = 4.683 \times 10^8$ lbf-ft-s/rad
Soil cross damping: $C_C = -5.044 \times 10^6$ lbf-s/rad
Distance of base to center of gravity: $h = 4.44$ ft

Dynamic forces at center of gravity of foundation system:
Unbalanced forces:
$F_v = 2000$ lbf (vertical) and $F_h = 2000$ lbf

(Lateral) dynamic moment:
$F_R = F_o \times (14\ \text{ft-h}) = 19{,}120$ lbf-ft

Frequency of force:
$\omega_o = 600\ \text{rpm} = 62.8\ \text{rad/s}$

Apply Eq. (6.4.3f) to calculate the complex vertical amplitude:
$A_V = F_v/[K_V - M \times \omega_o^2 + i \times C_V \times \omega_o] = (2000\ \text{lbf})/[2.629 \times 10^8\ \text{lbf/ft} - 14{,}982.9\ \text{lbf-s}^2/\text{ft} \times (62.8\ \text{rad/s})^2 + i \times 3.9 \times 10^6\ \text{lbf-s/ft} \times 62.8\ \text{rad/s}] = (4.012 \times 10^{-6} - i \times 4.826 \times 10^{-6})\ \text{ft}$

Apply Eq. (6.4.3g) to calculate the peak vertical amplitude:

$\Delta_V = \text{sqrt}[a^2 + b^2] = \text{sqrt}[(4.012 \times 10^{-6}\ \text{ft})^2 + (-4.826 \times 10^{-6}\ \text{ft})^2] = 0.075\ \text{mils}$

Vertical p-p displacement at the center of gravity of the machine:
$d_{PV} = 2 \times \Delta_V = 0.15\ \text{mils} < 1.5\ \text{mils}$ **OK**

Calculation for peak horizontal translation and rocking response as described in 6.4.3: The following equations are from Example 7 (6.7.7).
$B_1 = K_H - M \times \omega_o^2 + i \times C_H \times \omega_o = 2.467 \times 10^8\ \text{lbf/ft} - 14{,}982.9\ \text{lbf-s}^2/\text{ft} \times (62.8\ \text{rad/s})^2 + i \times 2.522 \times 10^6\ \text{lbf-s/ft} \times 62.8\ \text{rad/s} = (1.875 \times 10^8 + i \times 1.585 \times 10^8)\ \text{lbf/ft}$
$B_2 = K_C - K_H \times h + i \times C_C \times \omega_o\ i \times C_H \times h \times \omega_o = -4.933 \times 10^8\ \text{lbf/rad} - 2.467 \times 10^8\ \text{lbf/ft} \times 4.44\ \text{ft} + i \times (-5.044 \times 10^6\ \text{lbf-s/rad})(62.8\ \text{rad/s}) - i \times 2.522 \times 10^6\ \text{lbf-s/ft} \times 4.44\ \text{ft} \times 62.8\ \text{rad/s} = (-1.589 \times 10^9 - i \times 1.02 \times 10^9)\ \text{lbf/rad}$
$B_3 = K_R - I_R \times \omega_o^2 + K_H \times h^2 - 2 \times K_C \times h + i \times C_R \times \omega_o + i \times C_H \times \omega_o \times h^2 - i \times 2 \times C_C \times \omega_o \times h = 5.292 \times 10^{10}\ \text{lbf-ft/rad} - 763{,}281\ \text{lbf-s}^2\text{-ft} \times (62.8\ \text{rad/s})^2 + 2.467 \times 10^8\ \text{lbf/ft} \times (4.44\ \text{ft})^2 - 2 \times (-4.933 \times 10^8\ \text{lbf/rad}) \times 4.44\ \text{ft} + i \times 4.683 \times 10^8\ \text{lbf-ft-s/rad} \times 62.8\ \text{rad/s} + i \times 2.522 \times 10^6\ \text{lbf-s/ft} \times 62.8\ \text{rad/s} \times (4.44\ \text{ft})^2 - i \times 2 \times (-5.044 \times 10^6\ \text{lbf-s/rad}) \times 62.8\ \text{rad/s} \times 4.44\ \text{ft} = (5.915{\times}10^{10} + i \times 3.536 \times 10^{10})\ \text{lbf-ft/rad}$

$B_4 = B_1 \times B_3 - (B_2)^2 = (1.875 \times 10^8 + i \times 1.585 \times 10^8)$ lbf/ft × $(5.915 \times 10^{10} + i \times 3.536 \times 10^{10})$ lbf-ft/rad – $[(-1.589 \times 10^9 - i \times 1.02 \times 10^9)$ lbf/rad$]^2 = (4.008 \times 10^{18} + i \times 1.276 \times 10^{19})$ lbf^2

$A_H = (B_3 \times F_h - B_2 \times F_R)/B_4 = [(5.915 \times 10^{10} + i \times 3.536 \times 10^{10})$ lbf-ft/rad × 2000 lbf – $(-1.589 \times 10^9 - i \times 1.02 \times 10^9)$ lbf/rad × 19,120 lbf-ft$]/(4.008 \times 10^{18} + i \times 1.276 \times 10^{19})$ lbf$^2 = (9.765 \times 10^{-6} - i \times 8.583 \times 10^{-6})$ ft

Calculate the peak horizontal translation amplitude at center of gravity of foundation system:

$\Delta_H = \text{sqrt}[a^2 + b^2] = \text{sqrt}[(9.765 \times 10^{-6}\text{ ft})^2 + (-8.583 \times 10^{-6}\text{ ft})^2] = 0.156$ mils

$A_R = (B_1 \times F_R - B_2 \times F_h)/B_4 = [(1.875 \times 10^8 + i \times 1.585 \times 10^8)$ lbf/ft × 19,120 lbf-ft – $(-1.589 \times 10^9 - i \times 1.02 \times 10^9)$ lbf/rad × 2000 lbf$]/(4.008 \times 10^{18} + i \times 1.276 \times 10^{19})$ lbf^2 $= (5.131 \times 10^{-7} - i \times 3.688 \times 10^{-7})$ rad

Calculate the peak rocking amplitude at center of gravity of foundation system:

$\Delta_R = \text{sqrt}[a^2 + b^2] = \text{sqrt}[(5.131 \times 10^{-7}\text{ rad})^2 + (-3.688 \times 10^{-7}\text{ rad})^2] = 6.319 \times 10^{-7}$ rad

Lateral p-p displacement at the center of gravity of the machine:

$d_{PL} = 2 \times [\Delta_H + \Delta_R \times (14\text{ ft} - h)] = 2 \times [0.156\text{ mils} + 6.319 \times 10^{-7}\text{ rad} \times (14\text{ ft} - 4.44\text{ ft})] = 0.457\text{ mils} < 1.5$ mils **OK**

The previous three examples show that the rule-of-thumb method and the equivalent static load method are very useful for developing the initial size of dynamic equipment foundations or sizing of the foundations for dynamic equipment that are not highly vibration-sensitive. Among the three, the dynamic analysis method used in Example 3a and 3b is the only method that can confirm or evaluate whether the foundations sized on the basis of the first two methods will meet the frequency separation and allowable vibration limits.

7.10.5 *Example 4: Determination of pretension required in anchor bolts*—Problem description: A separate analysis has shown that each 2 in. (50 mm) diameter anchor bolt of ASTM A193/A193M, Grade B7, material for a compressor should carry a maximum horizontal dynamic load of 13,500 lbf (60 kN). Determine the tension (preload) required in the anchor bolt. Also, verify that the bolt size and bolt material selected are adequate.

Using a coefficient of friction of 0.12 and setting the contribution of compressor weight to zero, Eq. (7.4.3.1) gives the following minimum tension in the anchor bolt

$$T_{min} = 13{,}500\text{ lbf}/0.12 = 112{,}500\text{ lbf (500 kN)}$$

The recommended clamping force (lacking more explicit information) is 150 percent of this value, or 168,750 lbf (750 kN). The nominal area of a 2 in. (50 mm) diameter bolt is 3.14 in.^{2} (20.3 cm^2). With a yield stress of 105,000 psi (724 MPa), the yield force for the bolt is 330,000 lbf (1467 kN). The required force is 51 percent of the yield force, which is less than the 80 percent maximum and greater than the 15 percent minimum. The bolt should be tensioned to 168,750 lbf (750 kN) and that tension be maintained.

To calculate a minimum required yield stress for this application:

Bolt stress = 168,750 lbf/3.14 in.^{2} = 53,700 psi (375 MPa)

Required F_y = 53,700 psi/80% = 67,200 psi (463 MPa)

A material with a yield stress exceeding 67,200 psi (463 MPa) could be substituted for the ASTM A193/A193M, Grade B7, material.

CHAPTER 8—CONSTRUCTION CONSIDERATIONS

There are many construction considerations that are common to both dynamic equipment and static equipment. Various aspects of the construction of foundations for static equipment are addressed in ACI 351.2R. This chapter is limited in scope to address primarily the construction considerations that are pertinent only to dynamic equipment foundations. Engineers are advised to refer to ACI 351.2R for more information on construction.

8.1—Subsurface preparation and improvement

Results of a dynamic analysis sometimes demonstrate the advantage of modifying the soil stiffness dynamic parameters instead of increasing the size of the equipment foundation. Typical modifications include stiffening the soil by replacing part of the soil with engineered fill, lean concrete, or flowable fill. Other methods are vibro-compaction or rolling the soil. In some cases, the soil impedance is reduced by isolating the sides of the foundation from the soil.

CHAPTER 9—REPAIR AND UPGRADE

9.1—Overview of need for repair

9.1.1 *Introduction*—Existing concrete equipment foundations may need repair over the course of time because the machine is not functioning correctly, there is excessive downtime, or costs due to required maintenance. The unacceptable performance and high maintenance costs may be due to greater-than-anticipated loads, inadequate original design, inappropriate or inadequate maintenance, and other details provided in 9.1.2. Cracking and deterioration affecting the performance of the machine usually trigger foundation repairs. Cracks can cause anchor bolts to loosen and start a cycle of foundation deterioration. Furthermore, large cracks may allow fluids such as oil to penetrate the concrete and may stimulate cracking and concrete deterioration. The magnitude of repair needs to be assessed as described in 9.1.3. This involves conducting a root cause analysis to better understand the underlying problem, as presented in 9.1.4. Because concrete always cracks, it is important to evaluate whether the cracks present a cosmetic problem or a problem in machine performance as time progresses. A root cause analysis will help to uncover the real problems and focus on the most-effective approach to keep the machine in good operating condition for a long time.

9.1.2 *Causes of repair for a dynamic machine foundation*—A dynamic machine foundation may require repairs for numerous reasons such as:

a) Inadequate original design for actual operating conditions and dynamic forces

b) Change in machinery dynamic forces

c) The dynamic equipment has experienced loss of alignment or has degraded in tolerance from the specified vibration levels

d) Incorrect foundation width to height configuration

e) Improper attachment to the underlying concrete mat foundation or pile cap

f) Damage caused to the foundation by fire, impact, or explosion

g) Deterioration caused by aging

h) Excessive surface temperatures on concrete foundation and thermal cycling causing concrete degradation

i) Concrete and grout cracking

9.1.3 *Repair mitigation options*—An early assessment is required to decide the extent of repairs needed for the deteriorated foundation (ACI 364.1R; ACI 562). The repair plan should harmonize with the remaining service life of the equipment or planned future equipment for which it serves. Deciding the extent of repairs needed requires responding to the three critical options identified in the following.

9.1.3.1 *Restore foundation to original as-built condition*—Determine whether a repair procedure should simply return the foundation back to the original design or as-built condition. There are situations when simple repairs are appropriate (9.2.1). Provided the original as-built condition meets the current performance requirements for the equipment and could be durable within the environment, simply repairing the foundation to its preservice condition could be appropriate and cost-effective. Extension of design service life or desired upgrade of the foundation may not be practical for various reasons, such as the need to restore service quickly, budget constraints, and other factors. Sections 9.1.3.2 and 9.1.3.3 need, in general, more time and higher cost to repair a dynamic machine foundation. At times, restoring a foundation to as-built conditions could be the most optimum solution.

9.1.3.2 *Upgrade equipment foundation*—Determine whether an upgrade design should be commissioned and implemented to provide additional years of useful remaining service life for the dynamic equipment. Upgrades to the foundation occur when the performance of the as-built foundation is deemed insufficient for the current equipment performance requirements, leading to structural cracking and damage to its foundation, or will not be durable for the remaining service life mandated by the owner. To determine whether this option is appropriate, the engineer needs to understand the following, which is applicable to severely degraded foundations.

It has been observed that often the greatest deterioration from structural cracking, or other deterioration, is in the upper one-third of the concrete foundation where loads are greater and deterioration from process fluids or lubricating oil are more likely to be present. With proper engineering, the upgrade can be accomplished with the equipment left in place. In many cases, the extended remaining service life of an upgrade/repair can be 30 years, when the root cause of the equipment foundation deterioration is properly addressed.

If repair is needed because the original design was inadequate, the loads were increased, or any other significant reasons exist for repair, then incorporating upgrades into the repair design makes sense. It is undesirable to repair a foundation back to its original condition if operation has proven the original design to be inadequate.

9.1.3.3 *Replace equipment foundation*—Determine whether a completely new equipment foundation should be engineered and constructed to replace the existing foundation. The need for extensive repairs with a limited remaining service life, proactive equipment upgrades, and owner preferences typically prompt the replacement of a foundation. For reasons of construction safety, this option could require temporarily lifting the equipment out of the way. This approach could cost three to five times the cost of partial foundation removal and upgrade, and cause a much longer out-of-service time for the equipment. When plant outage (loss due to out of service) costs are estimated to be large, building new foundations for equipment may be considered as a viable alternative.

9.1.4 *Root cause analysis*—A root cause analysis is a better approach than simply addressing the symptoms, and helps in deciding which of the aforementioned three approaches should be implemented. This is illustrated with the following.

If deterioration of the equipment grouting results in increased vibration levels that exceed safe operating levels, simply replacing the grout, or choosing a different brand of grout, may not solve the vibrations issue for long if the wrong generic class of grout was originally chosen for the operating conditions of the equipment (ACI 351.1R).

If the concrete foundation has cracked badly, simply patching with epoxy injection or a repair mortar/concrete may not provide extended useful equipment foundation life, especially if the operating speed has changed, resulting in increased equipment dynamic forces. A repair upgrade with additional reinforcing steel and possibly post-tensioning may be needed to address the cracking and deterioration.

9.2—Discussion of repair options

9.2.1 *Simple repair situations*—If the root cause analysis determines that only a simple concrete repair is needed to bring the foundation back to the intent of the original design, guidance can be found in ACI 562 and ACI 546R. ACI 546R considers surface preparation as one of the most important steps in the repair of a concrete structure. It further states that the repair is only as good as the surface preparation. It provides enhanced guidelines for surface preparation that would be beneficial to adopt in a repair project.

Poor workmanship in the initial construction of a dynamic equipment foundation is the most likely situation where a simple repair is applicable. Various evaluation methods (ACI 228.2R) can provide valuable information about the condition of the concrete before removal. Quality control and assurance procedures (ACI 562) are required throughout the repair project to validate that the repair work complies with the design intent and specified performance criteria. Construction problems are often seen in the first 6 to 12 months of operation, whereas a problem due to poor design may take years to develop.

9.2.2 *Complex repair situations*—Often, the root cause analysis indicates that the dynamic forces are not being constrained and transmitted smoothly from the machine down through the foundation and into the soil below. When the symptoms are high vibration of the equipment, cracked concrete and grout, anchorage failure, or foundation rocking, the causes could be one or more of the following:

a) Incorrect original calculation of the projected magnitude of the dynamic forces, or a change in magnitude after a mechanical repair. Note that sometimes a very small change in a rotating part's weight can cause large increases in vibratory forces.

b) A change in operating speed can increase dynamic forces

c) A tall narrow foundation, made only as wide as the footprint of the machine

Often the minimum width and length supplied by the original equipment manufacturer (OEM) will have more rocking around the horizontal axis than a shorter, wider foundation. An OEM suggested concrete outline should only be a guide to minimum size. The OEM is not the foundation designer, and it is the engineer of record's responsibility to provide the best width-to-height ratio to help control rocking. Excessive rocking can be reduced in an existing foundation by adding vertical post-tension bolts (9.2.3(a)) and by adding foundation mass (9.2.3(e)). However, a revised vibration analysis with new parameters needs to be conducted to verify vibration levels and design allowable. When applicable, soil-structure interaction effects need to be included in the revised vibration analysis, as discussed in Chapter 5. Moreover, a parametric study would be beneficial to determine optimum values for bolt tension or additional foundation mass.

For any repair options, concrete foundation problems caused by equipment vibration can worsen over time as the equipment foundation deteriorates and causes an increase in equipment vibratory forces. These increased equipment forces may further deteriorate the dynamic equipment foundation with potential for further cracking of the concrete foundation and loosening of anchor bolts. This results in even further increases in vibratory forces. This cycle of degradation phenomenon continues as time progresses, demonstrating that an early and timely repair is always warranted for a dynamic machine foundation.

9.2.3 *Tools and techniques for machine foundation repair*—A variety of tools and techniques exist that may be applicable to the repair of a foundation that has deteriorated due to the problems described previously. The tools and techniques are described in the following:

a) Drill and install vertical post-tension bolts completely through the cracked sections and often down to an underlying concrete mat foundation or pile cap.

b) After partial or complete removal of the concrete and grout in the upper one-third of the foundation, replace it with a dense reinforcing bar grid of 1 percent (No. 6s [No. 19] on 6 in. [150 mm] centers in 6 in. [150 mm] vertical layers), and replace with a stronger concrete such as a steel fiber-reinforced polymer-modified concrete (PMC).

PMC (ACI 548.3R) is considered because the repair is being made, in most cases, from a degraded foundation or an inadequate original foundation design. A stronger concrete is beneficial in such situations. PMC is stronger in tensile and flexural strength, develops a better bond and adhesion to the original concrete, and develops less heat of hydration than ready mixed concrete.

Material selection for concrete repair is an important step. ACI 546R and ACI 546.3R provide guidelines for material selection of concrete repairs.

c) Upgrade the anchor bolts from typical 36,000 psi (248 MPa) lower-strength steel to higher-strength alloy steel, such as ASTM A193/A193M, Grade B7, with rolled threads. This material has minimum tensile strength of 125,000 psi (860 MPa) and minimum yield strength of 105,000 psi (720 MPa) for rods of various diameters (ACI 355.3R-11 Appendix A). The increased clamping force transfers the vibratory forces more efficiently into the foundation.

d) Add horizontal post-tensioning both ways in the upper one-third of the foundation. Earlier repair techniques used steel bolts to accomplish this, but newer techniques have been developed using closely spaced post-tensioned cables because of lower cost and ease of installation in a dense reinforcing bar grid. This technique is beneficial when severe vertical cracks exist.

e) Make changes to the concrete mass by increasing the horizontal width, length, or both, of the concrete where possible without equipment interference.

f) The connection between the equipment and the foundation is a critical point in providing a smooth path of the equipment vibratory forces down into the concrete foundation and into the soil or bedrock below. The improved anchorage suggested in 9.2.3(c) can help with increased clamping forces. Further, transmission of vibratory forces can be improved by changes such as using a poured polymer chock at each anchorage point in place of a full bed of grout, or one of the several adjustable steel or composite machinery supports that additionally correct for angularity and vertical alignment.

The aforementioned tools and techniques are generic suggestions; details depend on specific repair conditions, budget availability, significance of machine, desired design service life extension of the machine foundation, and feasible design details. It is advisable to verify design modifications for the repair job against applicable codes, standards, and specifications, and it should depend on the discretion of the engineer. Consider different options and costs associated with each of them and come to an optimum solution for the repair job.

CHAPTER 10—REFERENCES

Committee documents are listed first by document number and year of publication followed by authored documents listed alphabetically.

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ASA/ANSI S2.19:1999—Mechanical Vibration—Balance Quality Requirements of Rigid Rotors, Part 1: Determination of Permissible Residual Unbalance

American Concrete Institute

ACI 207.2R-07—Report on Thermal and Volume Change Effects on Cracking of Mass Concrete

ACI 215R-92(97)—Considerations for Design of Concrete Structures Subjected to Fatigue Loading
ACI 228.2R-13—Report on Nondestructive Test Methods for Evaluation of Concrete in Structures
ACI 307-08—Code Requirements for Reinforced Concrete Chimneys and Commentary
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ACI 350-06—Code Requirements for Environmental Concrete Structures and Commentary
ACI 351.1R-12—Report on Grouting Between Foundations and Bases for Support of Equipment and Machinery
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APPENDIX A—DYNAMIC SOIL PROPERTIES

Soil properties should be provided by a geotechnical engineer. Appendix A, however, provides engineers with a general overview of methods used to determine the various soil properties required for the dynamic analysis of machine foundations. Many references are available that provide a greater level of detail on the theory, standard practice, and factors that affect dynamic soil properties (Mitchell and Soga 2005; Seed and Idriss 1970; Stokoe et al. 1999; Andrus et al. 2003).

Many factors affect the ability of a given soil to support a dynamic machine foundation. These include excessive settlement caused by dynamic or static loads, liquefaction, expansive soils, and frost heave. All these should be addressed by a geotechnical engineer familiar with the local conditions.

The soil properties that are most important for the dynamic analysis of machine foundations are stiffness, density (ρ) and material damping (D_m). Stiffness properties are typically provided in the form of small strain Poisson's ratio (ν), and shear modulus (G), or alternatively in the form of shear (V_s) and compressional (V_p) wave velocities, as discussed in the following.

A.1—Poisson's ratio

Poisson's ratio (ν) is the ratio of transverse strain to longitudinal strain in the direction of applied force. In general, most soils have Poisson's ratio values in the range from 0.2 to 0.5. Laboratory determination of Poisson's ratio is difficult; therefore, it is sometimes estimated. Alternatively, for soils above the water table, ν is often determined, per Eq. (A.1), based on in-place measurements of the shear (V_s) and compressional (V_p) wave velocities.

$$\nu = \frac{1}{2}\left(\frac{(V_p/V_s)^2 - 2}{(V_p/V_s)^2 - 1}\right) \quad \text{(A.1)}$$

The dynamic response of a foundation system is generally insensitive to variations of Poisson's ratio in the range of values common for dry or partial saturated soils (that is, 0.25

to 0.4). On the other hand, the dynamic vertical and rocking responses of a foundation are very sensitive to Poisson's ratios greater than 0.4. Therefore, when calculating this type of foundation response, it should be taken into account that the Poisson's ratio of a fully saturated soil, under dynamic or undrained load conditions, approaches 0.5.

If the Poisson's ratio value is not provided in the geotechnical report, then values from 0.25 to 0.40 should be used for cohesionless granular (free-draining) soils, and from 0.33 to 0.5 for cohesive (impermeable) soils. In addition, a value of or near 0.5 should be used for fully saturated soils below the water table; although a slightly lower number—for example, 0.48—is sometimes necessary for stability of numerical calculations.

A.2—Dynamic shear modulus

Dynamic shear modulus (G) is the most important soil parameter influencing the dynamic behavior of the machine-foundation-soil system. The dynamic shear modulus represents the slope of the shear stress versus shear strain curve. Most soils do not respond elastically to shear stress but rather with a combination of elastic and plastic strain. For that reason, plotting shear stress versus shear strain results in a curve, not a straight line, as shown in Fig. A.2. Thus, the value of G varies depending on the strain magnitude: the lower the strain, the higher the dynamic shear modulus. In general, for design of machine foundations, the initial tangent shear modulus (G_{max}) is commonly used and recommended. However, due to inherent variations of the G_{max} values of in-place soils, completing more than one dynamic foundation analysis is recommended. For this purpose, the geotechnical engineer should recommend a best estimate (BE), lower bound (LB), and upper bound (UB) for the shear modulus. If not provided, the UB and LB values of the shear modulus can be determined from the BE values per Eq. (A.2a) and (A.2b), where COV is the coefficient of variation. In particular, per USNRC (2013), it is recommended to take the COV as 0.5 for well-investigated sites and at least 1.0 for sites that are not well investigated, that is, lacking the required number of borings and geophysical tests per the applicable jurisdiction.

$$G_{LB} = \frac{G_{BE}}{1 + \text{COV}} \quad \text{(A.2a)}$$

$$G_{UB} = G_{BE}(1 + \text{COV}) \quad \text{(A.2b)}$$

As discussed in the following section, several methods are available for obtaining design values of the G_{max} modulus; these include:

a) Field measurements of the shear wave velocity of the in-place soils

b) Laboratory tests on soil samples

c) Correlation to other soil properties

A.2.1 *Field determination*—Field measurements are the most common method for determining the G_{max} modulus of a given soil. These methods involve measuring the

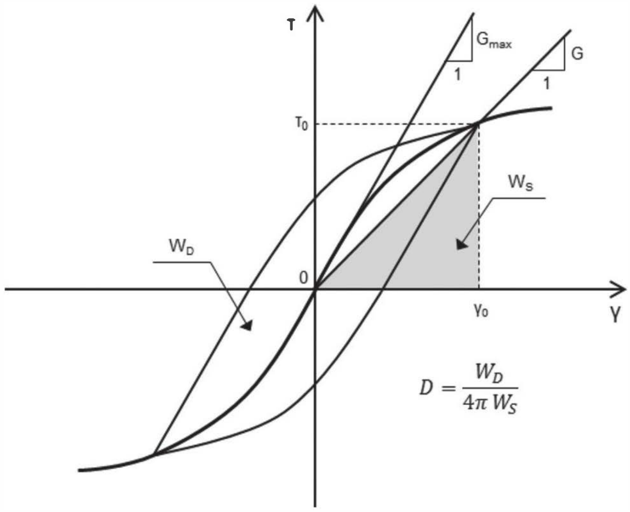

Fig. A.2—Hysteresis loop for one cycle of loading showing G_{max}, G, and D.

compression (V_P), shear (V_S), Rayleigh (V_r), or all of these wave velocities of the in-place soil, at the actual foundation location(s). The methods available for measuring wave velocities of the in-place soil include:

a) The cross-hole method

b) The down-hole method

c) The up-hole method

d) Seismic cone penetration test (SCPT)

e) Seismic refraction

f) Spectral analysis of surface wave (SASW)

g) Multichannel analysis of surface waves (MASW)

h) Refraction microtremor (ReMi) method

A detailed discussion of the aforementioned methods is outside the scope of this report; for specific details, refer to Fang (1991) and Stokoe et al. (1999).

A.2.2 *Laboratory determination*—Laboratory tests are considered less accurate than field measurements for calculating the shear modulus, due to the possibility of sample disturbance.

The most common laboratory test is the resonant-column method, where a cylindrical sample of soil is placed in a device capable of generating forced torsional vibrations. The soil sample is excited at different frequencies until the resonant frequency is determined. The dynamic soil modulus can be calculated based on the resonant frequency, length, density, and end conditions of the soil sample. Specific details are provided in ASTM D4015, which defines the resonant-column method.

A.2.3 *Correlation to other soil properties*—Empirical correlations can be used for estimating the shear modulus of the soil. However, the engineer should be especially careful when using any correlation method because they offer, at the best order of magnitude, estimated values. Correlations methods should only be used for preliminary design or for analysis of small noncritical foundations with small dynamic loads. Correlation of the dynamic shear modulus to other soil properties should be considered as providing a range of possible values, not providing a single exact value. This should be accounted for in the preliminary design by applying Eq. (A.2a) and (A.2b) with a COV between 2 and 3.

A.2.3.1 *Correlations for sands and gravels*—Hardin and Richart (1963) determined that the soil void ratio (e) and the in-place mean effective stress (σ_o) had the most impact on the dynamic shear modulus. In general, G_{max} decreases with an increase of void ratio and increases with the confining stress and over-consolidation ratio. Accordingly, Hardin and Richart (1963) and Hardin and Black (1968) developed the following relationships for estimating the dynamic shear modulus G_{max} of round-grained sands with e between 0.3 and 0.8, where $\sigma_o = (\sigma_1' + \sigma_2' + \sigma_3')/3$ for laboratory samples, and $\sigma_o = \sigma_1'(1 + 2K_0)/3$ for in-place soils:

$$G_{max} = \frac{144{,}000(2.17-e)^2\sqrt{\sigma_0}}{1+e} \text{ (lb/ft}^2\text{)}$$
$$G_{max} = \frac{6{,}900{,}000(2.17-e)^2\sqrt{\sigma_0}}{1+e} \text{ (Pa)} \quad \text{(A.2.3.1a)}$$

Similarly, for angular-grained materials with e between 0.6 and 1.3, the dynamic shear modulus can be estimated from

$$G_{max} = \frac{68{,}300(2.97-e)^2\sqrt{\sigma_0}}{1+e} \text{ (lb/ft}^2\text{)}$$
$$G_{max} = \frac{3{,}270{,}000(2.97-e)^2\sqrt{\sigma_0}}{1+e} \text{ (Pa)} \quad \text{(A2.3.1b)}$$

Additionally, Eq. (A.2.3.1c) proposed by Seed and Idriss (1970) and further refined by Seed et al. (1986) can be used for correlating the dynamic shear modulus to the relative density and confining pressure of granular soils. In this equation, $(K_2)_{max}$ is a correlation coefficient and n_G is typically taken as 0.5. Table A.2.3.1a provides the variation of $(K_2)_{max}$ with respect to relative density and void ratio, whereas Table A.2.3.1b, from Seed et al. (1986), provides examples of the values of $(K_2)_{max}$ of sandy soils and gravelly soils determined at various sites, for depths ranging from 10 to 300 ft (3 to 91 m). For additional correlations and discussion, refer to Mitchell and Soga (2005).

$$G_{max} = 1000(K_2)_{max}(\sigma_0)^{n_G} \text{ (lb/ft}^2\text{)}$$
$$G_{max} = 6920(K_2)_{max}(\sigma_0)^{n_G} \text{ (Pa)} \quad \text{(A.2.3.1c)}$$

A.2.3.2 *Correlations for clays*—For normally consolidated clays with low surface activity and e between 0.5 and 1.7, the dynamic shear modulus may be estimated using Eq. (A.2.3.1b). For over-consolidated clays, Hardin and Drnevich (1972) proposed Eq. (A.2.3.2) for G_{max}, where $e < 2.0$ and OCR is the over-consolidation ratio; M is an exponent ranging between 0 and 0.5, depending on the plasticity index detailed in Table A.2.3.2; N ranges from 0.51 to 0.73 for most sandy and clay soils, but 0.65 is recommended; and p_a is a reference atmospheric pressure. As already mentioned, additional correlations have been reported (Mitchell and Soga 2005).

Table A.2.3.1a—Values of K_2 coefficient for various void ratios and relative densities (Seed and Idriss 1970)

e	K_2	D_r, %	K_2
0.45	70	30	34
0.5	60	40	40
0.6	51	45	43
0-.7	44	60	52
0.8	39	75	59
0.9	34	90	70

$$\frac{G_{max}}{p_a} = \frac{321(2.97-e)^2}{1+e}\text{OCR}^M\left(\frac{\sigma_0}{p_a}\right)^N \quad \text{(A.2.3.2)}$$

A.3—Soil damping

The material damping ratio D_m represents the energy dissipated by the soil during the vibrations induced by the machine-foundation system. Mechanisms that contribute to material damping are friction between soil particles, strain rate effects, and nonlinear soil behavior. Referring to Fig. A.2, the material or hysteretic damping is defined by Eq. (A.3a), where W_D is the energy dissipated in one cycle of loading and W_S is the maximum strain energy stored during the cycle. The area inside the hysteresis loop is W_D and the area of the triangle is W_S.

$$D = \frac{W_D}{4\pi W_s} \quad \text{(A.3a)}$$

The material damping is frequency-independent and, thus, cannot be modeled using a viscous dashpot. Instead, the complex shear modulus (G^*) defined by Eq. (A.3b) is typically used for all dynamic calculations involving material damping.

$$G^* = G(1 + 2iD_m) \quad \text{(A.3b)}$$

The material damping increases with the shear strain; however, for properly designed machine foundations, the low strain or minimum material damping (D_{min}) should be used.

Minimum damping D_{min} is measured in the laboratory using torsional shear (TS) or resonant column (RC) tests. However, the problem with laboratory measurements of D lies in the identification of equipment-related energy losses as discussed by Stokoe et al. (1999). This effect should be deducted from the measured values to obtain the correct damping value D. In addition, it has been suggested that the soil damping D should be measured at frequencies and number of loading cycles similar to those of the anticipated cyclic loading (for example, seismic or machine vibration) to account for the effects of these factors. In place of experimental values, the low-strain minimum soil damping ratio can be set to 1 percent or less. For additional discussion on

Table A2.3.1b—Values of $(K_2)_{max}$ of sandy and gravelly soils (Seed et al. 1986)

	Soil description	Location	Depth, ft (m)	$(K_2)_{max}$
Sandy soils	Loose moist sand	Minnesota	10 (3)	34
	Dense dry sand	Washington	10 (3)	44
	Dense saturated sand	So. California	50 (15)	58
	Dense saturated sand	Georgia	200 (60)	60
	Dense saturated silty sand	Georgia	60 (18)	65
	Dense saturated sand	So. California	300 (90)	72
	Extremely dense silty sand	So. California	125 (38)	86
Gravel soils	Sand, gravel, and cobbles with little clay	Caracas	200 (60)	90
	Dense sand and gravel	Washington	150 (45)	122
	Sand, gravel, and cobbles with little clay	Caracas	255 (77)	123
	Dense sand and sandy gravel	So. California	175 (53)	188

Table A.2.3.2—Values of M coefficient for clays (Hardin and Drnevich 1972)

Plasticity index	M
0	0.00
20	0.18
40	0.30
60	0.41
80	0.48
≥100	0.50

this subject, refer to Stokoe et al. (1999) and Mitchell and Soga (2005).

A.4—Radiation damping

Radiation damping or geometry damping is a term used to describe the dissipation of elastic energy that takes place for a foundation vibrating in an elastic half-space, even when the half-space has no material damping.

To illustrate this phenomenon, consider the system shown in Fig. 5.3.2. The vibration of the foundation results in elastic waves (body and surface waves) with significant energy content. In a half-space, these waves propagate continuously away from the structure, as shown in Fig. 5.3.2. Therefore, the energy carried by the waves is dissipated (lost) in the half-space medium, giving rise to attenuation of the vibration (that is, damping). In summary, the term "radiation damping" is used to characterize a mechanism where energy seems to be lost due to wave propagation. Unlike the material damping, radiation damping is frequency-dependent and is generally modeled using a damper/dashpot.

In the case of a foundation supported on a soil layer overlying a rigid stratum, the radiation damping (not the material damping) vanishes for vibration frequencies below a threshold or cutoff frequency, which is a function of the vibration mode and soil properties. In the case of horizontal vibration, the cutoff frequency corresponds to the fundamental frequency (f) of the soil layer in Eq. (A.4), where V_s is the shear wave velocity of the layer and H is its thickness.

$$f = \frac{V_s}{4H} \tag{A.4}$$

Additionally, the radiation damping also vanishes for half-spaces with rapidly increasing stiffness (Wolf and Song 2002).

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