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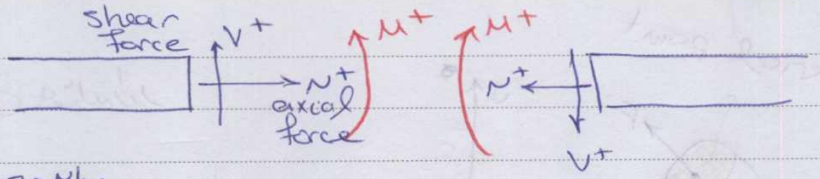
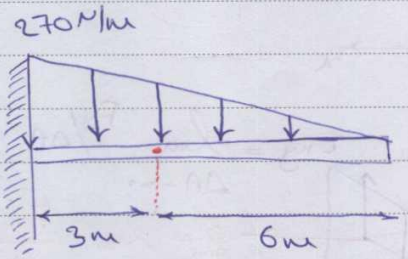
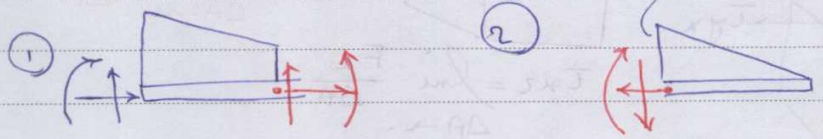
* مقاومت مصالح *



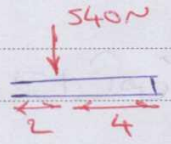


مثال:

$\frac{w}{6} = \frac{270}{9} \Rightarrow w = 180N$



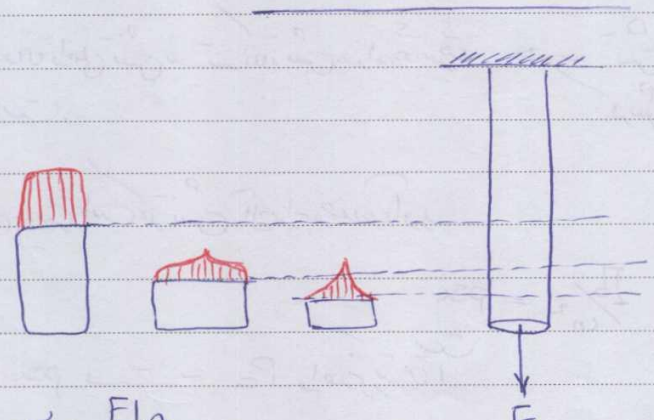
راحت تر ②



$N = 0$

$V = 540N$

$M = -1080 \, Nm$



$\sigma = F/A$

normal stress

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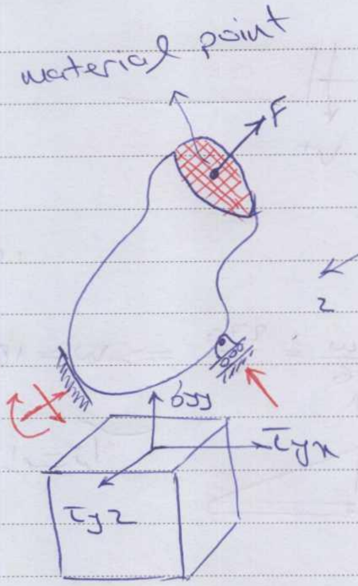
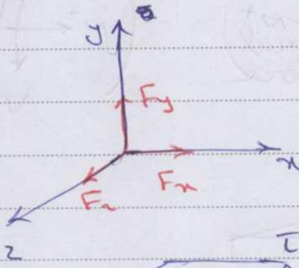
σ تنش عمودی

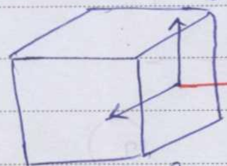
τ تنش برشی





تنش ۶ مؤلفه دارد

$$\tau_{xy} = \lim_{\Delta A \to 0} F_y/\Delta A$$



$$\sigma_{xx} = \lim_{\Delta A \to 0} \frac{F_x}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{F_z}{\Delta A}$$

نقطه مادی آب از آهن بزرگتر است چون چگالی آب کمتر از آهن است.

material point کوچکترین نقطه جسم است که خواص جسم را به همراه دارد

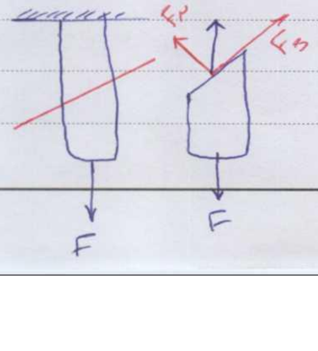
stress تنش ⟹ خود نیروهای داخلی تنش نیستند بلکه تنش مؤلفه‌های نیروی داخلی است.

strain کرنش

کرنش چیزی است که تنش آن را به وجود آورد.

$N/m^2 = Pa$ $\quad lb/in^2 = psi$

psi نسبت به Pa واحد بزرگتری است



$\sigma = \frac{F_n}{A_0}$ تنش عمودی

$\tau = \frac{F_s}{A_0}$ تنش برشی

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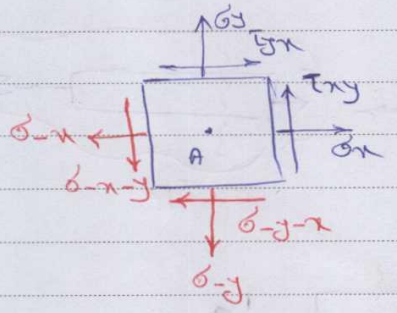
$$\text{تنش} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

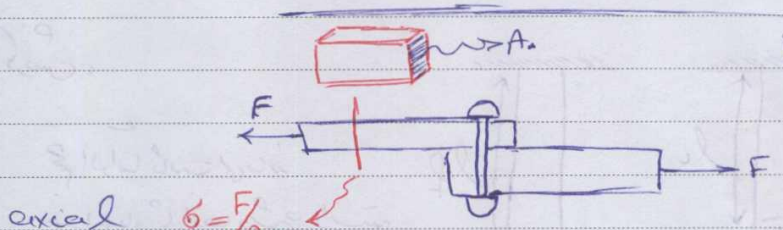
مولفه‌های تنش در حالت کلی (3 بعدی)

در حالت دوبعدی تنها صفحه‌ی کاغذ

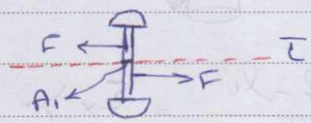


$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum (M_z)_A = 0 \ * \end{cases} \Rightarrow \tau_{xy} = \tau_{yx}$$

$$\text{تنش} = \begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix}$$



bearing stress
F/td



single shear
shearing stress
$\tau = F/A.$

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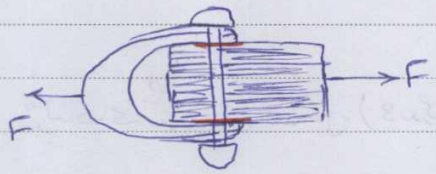
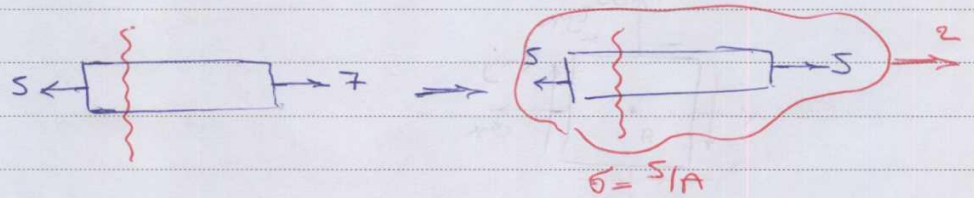
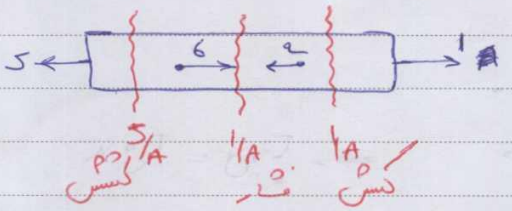
double shear

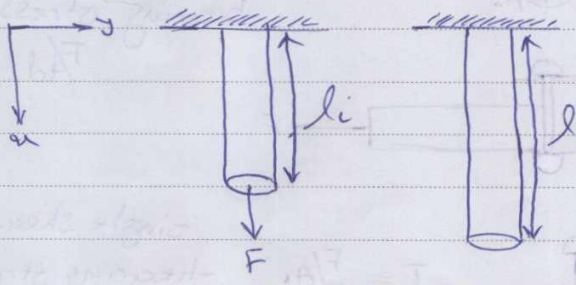


$$\tau = \frac{F/2}{A} = \frac{F}{2A}$$





تنش مجاز (allowable stress) = تنش تسلیم (کاری) / safty factor



$$\text{strain} = \frac{l_f - l_i}{l_i} = \varepsilon_x > 0$$

طول میله زیاد می شود

$$\frac{d_f - d_i}{d_i} = \varepsilon_y < 0$$

قطرمیله کم می شود

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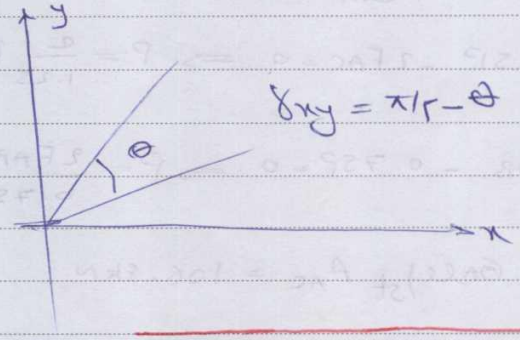
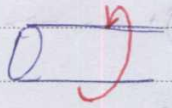
کلاس حل تمرین 12:30-13:30 سه‌شنبه

تنش = $\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$ 3 بعدی

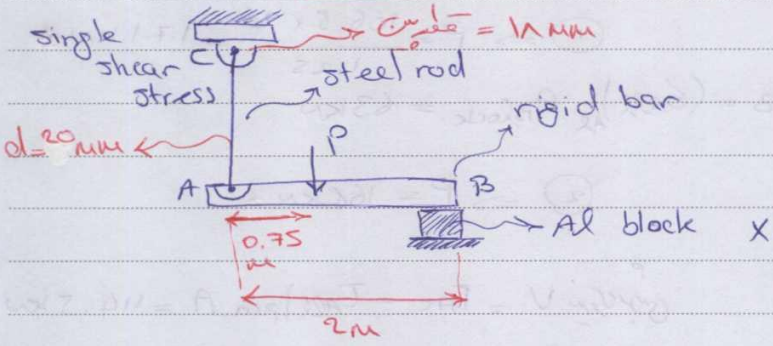
تنش = $\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$ 2 بعدی



$\gamma_{xy} = \pi/2 - \theta$



مثال:



$XS_{Al} = 1800\ mm^2$

$\sigma_F)_{st} = 680\ MPa$ (Failure)

$P_{MAX} = ?$ مطلوب

$\sigma_F)_{Al} = 70\ MPa$

$S.F = 2$

$\tau_F)_{pin} = 900\ MPa$

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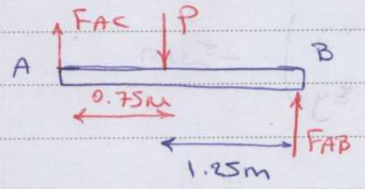
$\sigma_{All})_{st} = \frac{680}{2} = 340\ MPa$ $\quad \sigma_{All})_{Al} = 35\ MPa$

$\tau_{All})_{pin} = 450\ MPa$



$\sum M_B \circlearrowleft + = 0 \Rightarrow 1.25P - 2F_{AC} = 0 \Rightarrow P = \frac{2}{1.25} F_{AC}$ (1)

$\sum M_A \circlearrowleft + = 0 \Rightarrow 2F_{AB} - 0.75P = 0 \Rightarrow P = \frac{2F_{AB}}{0.75}$ (2)

"Rod AC" $\quad F_{AC} = \sigma_{All})_{st} A_{AC} = 106.8\ kN$

(1) $\Rightarrow P = \frac{106.8 \times 2}{1.25} = 171\ kN$ *

"Block Al" $\quad F_{AB} = (\sigma_{All})_{Al} A_{Block} = 63\ kN$

(2) $\Rightarrow P = 168\ kN$ *

Pin "A" or "C" $\quad$ نیروی برشی $V = F_{AC} = \tau_{All})_{pin} A = 114.5\ kN$

(1) $\Rightarrow P = \frac{11.4(2)}{1.25} = 183\ kN$ *

* $\Rightarrow P_{max} = 168\ kN$

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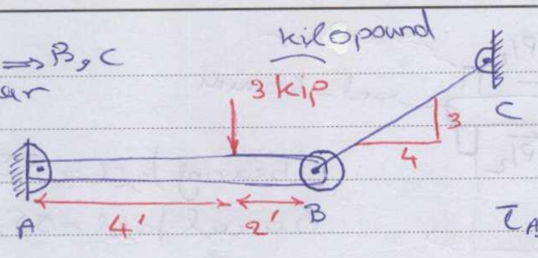
مثال:

single shear ⟹ B, C

A ⟹ double shear



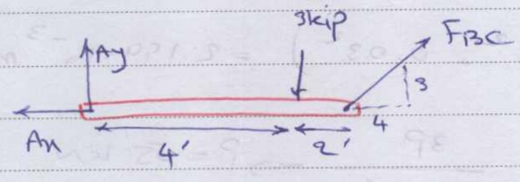
' ft

" inch

$\tau_{All})_{pin} = 12.5$ ksi $\left(\frac{\text{kpound}}{\text{sq. inch}}\right)$

$\sigma_{All})_{CB} = 16.2$ ksi

کوچکترین قطر پین و قطر میله BC را تعیین کنید تا $\left(\frac{1}{16}''\right)$



* دقت: برشی در نقطه A به پین وارد می‌شود ناشی کل نیروی A است نه فقط مؤلفه x نیروی A

$\sum M_B \circlearrowleft + = 0 \Rightarrow 6A_y = 3 \times 2 \Rightarrow A_y = 1$ kip

$\sum M_A \circlearrowleft + = 0 \Rightarrow 3 \times 4 = 6B_y \Rightarrow B_y = 2$ kip

$F_{BC} = 5/3\, B_y = 3.33$ kip

$A_x = 4/5\, F_{BC} = 2.67$ kip

$V_A = \sqrt{1^2 + (2.67)^2} = 2.85$ kip

$A_A = \dfrac{2.85/2}{\tau_{All}} \Rightarrow d_A = 0.381''$

$A_C = \dfrac{3.33}{12.5} \Rightarrow d_B = 0.625''$

$A_{BC} = \dfrac{F_{BC}}{\sigma_{All})_{BC}} = \dfrac{3.33}{16.2} \Rightarrow d_{BC} = 0.512''$

choose $0.5625''$

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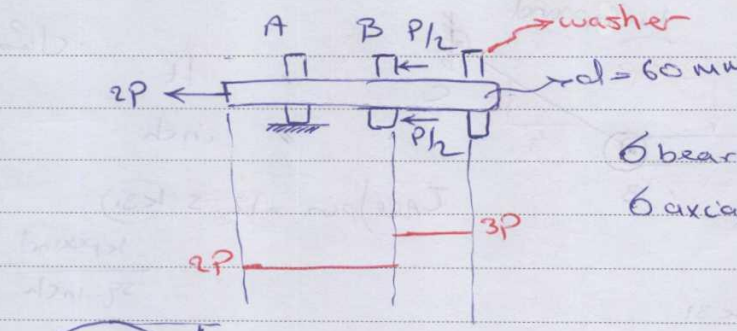
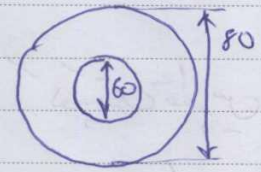


due

$\sigma_{bearing\,all} = 75\ MPa$

$\sigma_{axial\,all} = 55\ MPa$



$$55\times10^{6} = \frac{3P}{\pi(0.03)^2} \Rightarrow P = 51.8\ kN$$

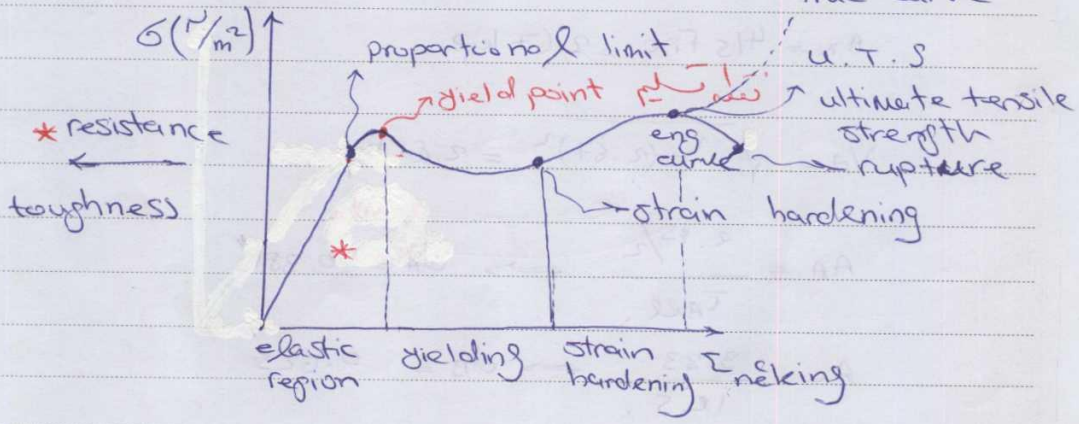
$$A_b = \pi(0.04^2 - 0.03^2) = 2.199\times10^{-3}\ m^2$$

$$2.199\times10^{-3} = \frac{3P}{75\times10^{6}} \Rightarrow P = 55\ kN$$

$$P_{max} = 51.8\ kN$$

---

ductile materials



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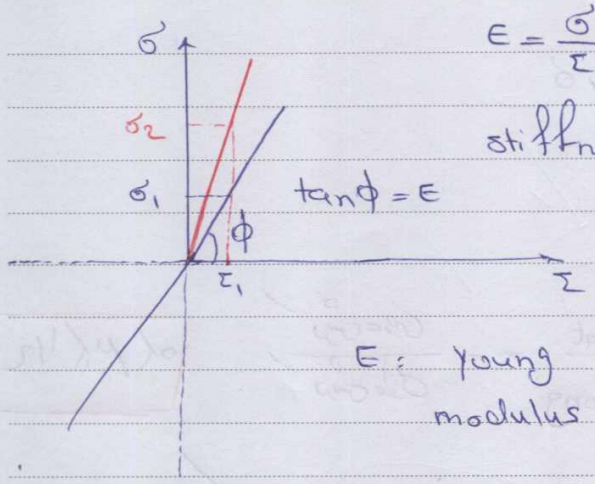


$E = \frac{\sigma}{\varepsilon} \rightarrow \boxed{\sigma = E\varepsilon}$ Hooke's Law

stiffness

E : Young modulus
modulus of elasticity

$E \equiv \frac{\sigma}{\varepsilon}$ N/m^2

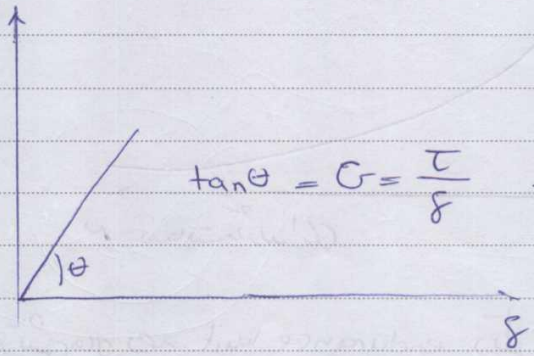
$E_c = E_t$ ← برای اکثر مصالح

strain energy ← انرژی مصرف شده برای تغییر حجم ΔV

$\Delta U = \frac{1}{2}\sigma\varepsilon\Delta V = \frac{1}{2}F/L^2 \cdot L/L \cdot L^3 = F.L$

σ ← انرژی

انرژی کرنشی واحد حجم $= \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\sigma\frac{\sigma}{E} = \frac{1}{2}\frac{\sigma^2}{E}$ (سطح زیر نمودار σ-ε)



$\tan\theta = G = \frac{\tau}{\gamma} \rightarrow \boxed{\tau = G\gamma}$

shear elastic modulus

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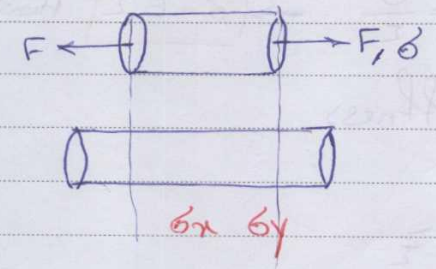


ضریب پواسون — Poisson ratio

$$\mu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{\text{کرنش عرضی}}{\text{کرنش طولی}}$$

$0 < \mu < 1/2$

برای مواد غیرقابل تراکم (حجم ثابت) $\mu = 1/2$

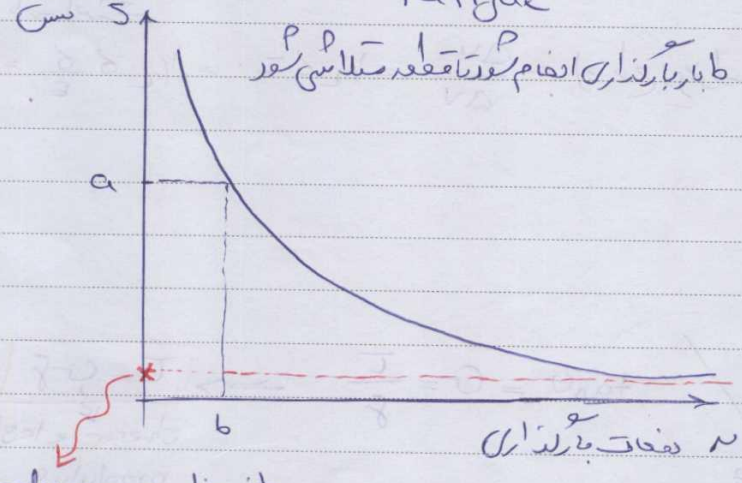
$$G = \frac{E}{2(1+\mu)}$$

Fatigue

نمودار تنش S — تعداد دفعات بارگذاری N

بارگذاری تنش a باید b بار بارگذاری انجام شود تا قطعه متلاشی شود

* S-N curve *



Endurance limit

برای بیشتر قطعات تنش‌های زیر endurance limit دیگر گرفته نمی‌شود

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(Creep خزش) کرنش

تنش ثابت

نمودار برای تنش رسم شده

کرنش بدست آمده براساس قانون هوک

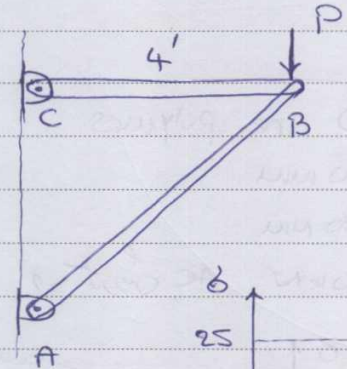
$\varepsilon = \frac{1}{E}\sigma$

زمان

کرنش

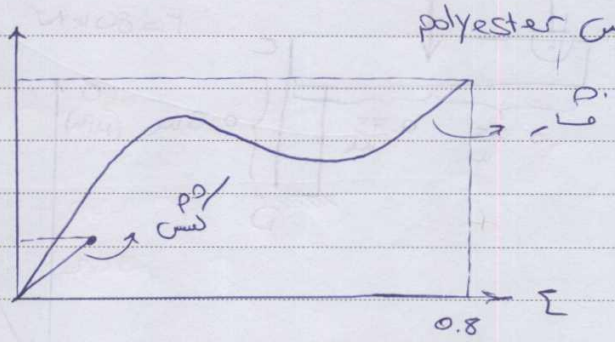
زمان

مثال:



AB سطح $= 1.5 inch^2$

BC سطح $= 4 inch^2$



از جنس polyester

P_{max} قبل از failure: $P_{max} = ?$

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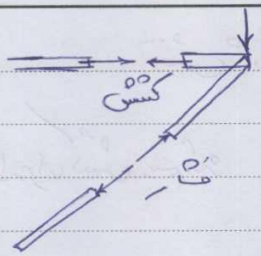


$\uparrow \sum F_y = 0 \Rightarrow \frac{3}{5} F_{AB} - P = 0$

$\Rightarrow F_{AB} = 1.667P$

$\sum F_x = 0 \Rightarrow \frac{4}{5} F_{AB} = F_{BC}$

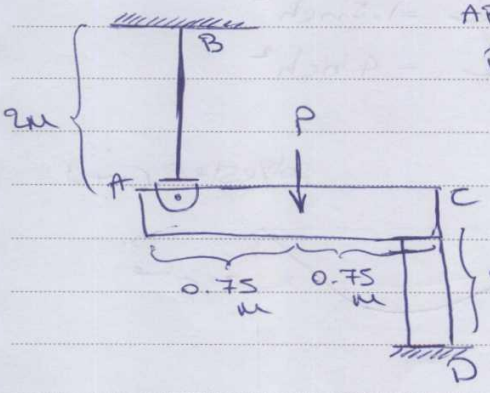
$\Rightarrow F_{BC} = \frac{4}{5}(1.667P) = 1.333P$

$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} \Rightarrow 5 = \frac{F_{BC}}{4} \Rightarrow F_{BC} = 20 \text{ kip}$

$\Rightarrow P = \frac{20}{1.333} = 15 \text{ kip}$

$25 = \frac{F_{AB}}{A_{AB}/1.5} \Rightarrow F_{AB} = 37.5 \text{ kip} \Rightarrow P = \frac{37.5}{1.667} = 22.5 \text{ kip}$

$\Rightarrow \boxed{P_{MAX} = 15 \text{ kip}}$

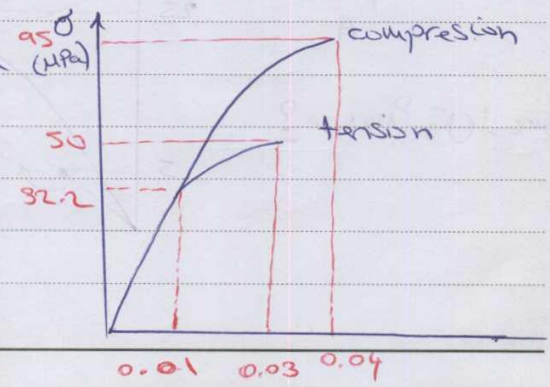


AB & CD are polymers

$D_{AB} = 60 \text{ MM}$

$D_{CD} = 80 \text{ MM}$

$P = 80 \text{ kN}$ AC تغیر P =؟



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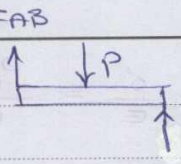


$F_{AB} = F_{CD} = P/2 = 40kN$

$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40\times10^3}{\frac{\pi}{4}(0.04)^2} = 31.83$ MPa $<$ 32.2 MPa $= \sigma_{yield}$

استفاده کردیم $\sigma = E\varepsilon$ هستیم در ناحیه الاستیک یعنی proportional limit

$E = \frac{32.2(10^6)}{0.01} = 3.22$ GPa

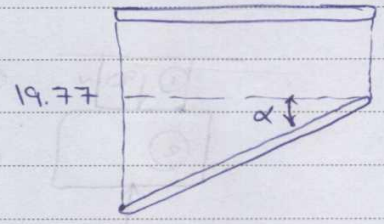
$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83\times10^6}{3.22\times10^9} = 0.0098$

$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0098 \times 2000 = 19.77$ mm

$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40\times10^3}{\frac{\pi}{4}(0.08)^2} = 7.958$ MPa in the proportional limit.

$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958\times10^6}{3.22\times10^9} = 0.00247$

$\delta_{CD} = \varepsilon_{CD} L_{CD} = 0.00247 \times (500) = 1.236$ mm



$\tan\alpha = \frac{19.77 - 1.236}{1500}$

$\Rightarrow \alpha = 0.708°$

Subject:

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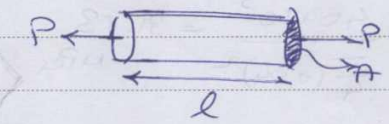
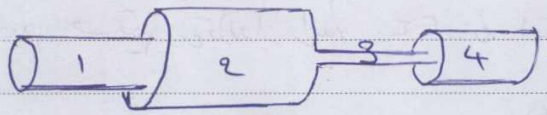
$$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E} = \frac{P/A}{E} \Rightarrow \frac{\Delta l}{l} = \frac{P/A}{E}$$

$$\Rightarrow \Delta l = \frac{Pl}{AE}$$



$$\Delta l = \sum_{i=1}^{N} \frac{P_i l_i}{A_i E_i}$$

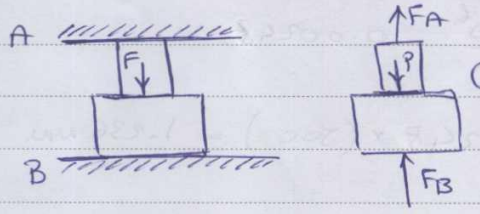


$$\Delta l = \int_0^l \frac{F(x)\,dx}{A(x)\,E(x)}$$

E معمولاً به صورت تابعی از x نیست بلکه مثلاً قطعه‌ای از 3 جنس مختلف

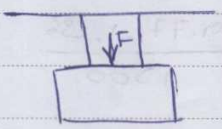
$$\sum_{i=1}^{3} \frac{1}{E_i} \int_0^{l_i} \frac{F(x)\,dx}{A(x)}$$

مثال:

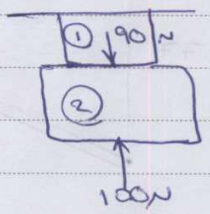


از تعادل استاتیکی داریم $F_A + F_B = P$

سازگاری تغییر شکل $\Delta l_1 + \Delta l_2 = 0$

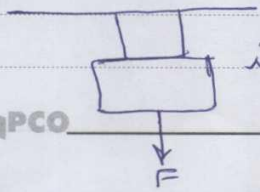


بالایی کشیده می‌شود
پایینی تغییری نمی‌کند



تنش $\sigma_1 = \frac{10}{A_1}$

تنش $\sigma_2 = \frac{100}{A_2}$



هردو کشیده می‌شوند

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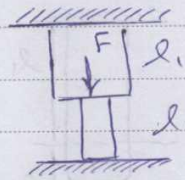
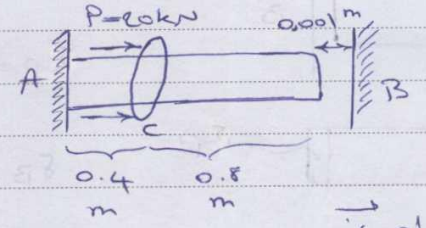


$\sum F_y = 0 \Rightarrow R_A - F + R_B = 0 \quad (1)$

$\delta = \frac{R_A l_1}{A_1 E_1} - \frac{R_B l_2}{A_2 E_2} = 0 \quad (2)$

(1), (2) $\Rightarrow R_A$, R_B

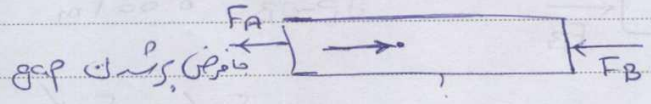
مثال:



$E = 200 GPa$

$d = 0.005 m$

راه حل: تغییر طول AB را حاصل جمع تغییر طول AC و CB در نظر می‌گیریم. سپس با توجه به اینکه تغییر طول کل از 0.001 بزرگتر است یا کوچکتر، فرض پرشک gap را انتخاب می‌کنیم.



$F_A + F_B = 20 \times 10^3 N \quad (1)$

$\Delta l = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{BC}}{AE} = 0.001$

$\Rightarrow 0.4 F_A - 0.8 F_B = 3927 \quad (2)$

(1), (2) $\Rightarrow F_A = 16.6 kN$

$F_B = 3.39 kN$

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فرض پرشک gap درست بوده چون F_B مثبت شده

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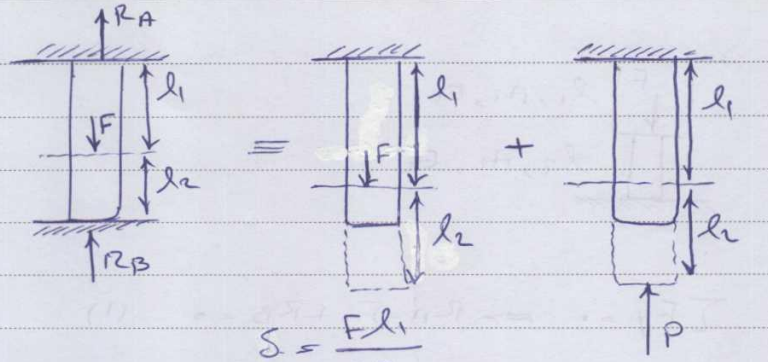
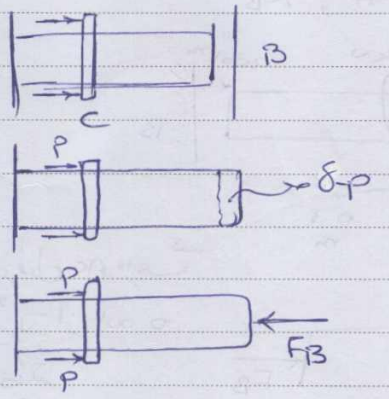


$$\delta = \frac{F l_1}{AE}$$

$$\delta = \frac{P(l_1+l_2)}{AE}$$

$$\frac{F l_1}{AE} = \frac{P(l_1+l_2)}{AE}$$

مثال: A, B, C



$$\delta_P = \frac{P L_{AC}}{AE} = 0.002\,m$$

$$\delta_B = \frac{F_B L_{AB}}{AE}$$

$$\delta_P - \delta_B = 0.001\,m$$

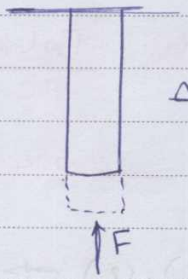
$\Rightarrow F_A$ ✓ , F_B ✓

$$\delta_T = \alpha(\Delta T) l$$

$$\delta_F = \frac{F l}{AE}$$

$$\alpha(\Delta T) l = \frac{F l}{AE}$$

$$\Rightarrow F = \alpha \Delta T A E$$



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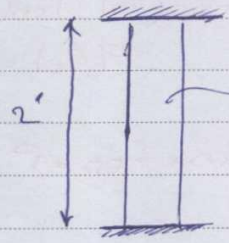
: due



$XS = (0.5)^2$

$T_1 = 60\ ^\circ F \quad T_2 = 120\ ^\circ F$

$$F = 6.6\times10^{-6}\ \frac{1}{^\circ F}(120-60)^\circ F(0.5)^2 \times 29\times10^3$$

$$= 2.87\ \text{kip}$$

$$\sigma = \frac{2.87}{(0.5)^2} = 11.5\ \text{ksi}$$

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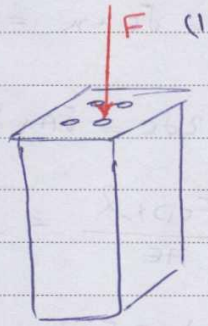
composites

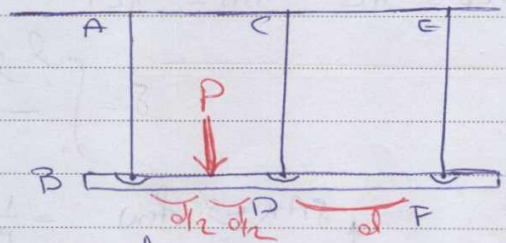


(1) $F = f_B + f_S \qquad \delta_B = \delta_S$

(2) $\dfrac{f_B \ell}{A_B E_B} = \dfrac{f_S \ell}{A_S E_S}$

(1), (2) $\Rightarrow f_B$ ✓, f_S ✓

: due



bars made of same material with same XS

find the forces in each bar

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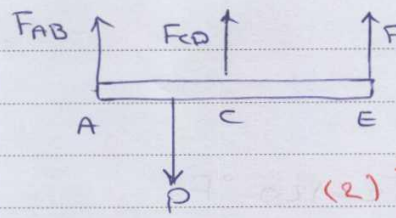
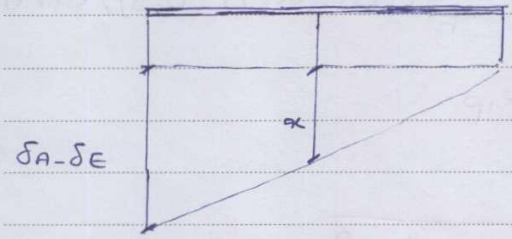


(1) $\sum M_A = 0 \Rightarrow F_{EF}(2d) + F_{CD}(d) - P(d/2) = 0$

(2) $\sum F_y = 0 \Rightarrow F_{AB} + F_{CD} + F_{EF} = P$



$$\frac{x}{d} = \frac{\delta_A - \delta_E}{2d} \Rightarrow x = \tfrac{1}{2}(\delta_A - \delta_E)$$

$$\delta_C = \delta_E + x = \frac{\delta_A + \delta_E}{2}$$

$$\Rightarrow 2\delta_C = \delta_A + \delta_E$$

$$\Rightarrow \frac{2F_{CD}\times l}{AE} = \frac{F_{AB}\times l}{AE} + \frac{F_{EF}\,l}{AE}$$

$$\Rightarrow (3)\ 2F_{CD} = F_{AB} + F_{EF}$$

$$F_{CD} = P/3 \qquad F_{EF} = P/12 \qquad F_{AB} = 7/12\,P$$

$\delta = \frac{\delta l^2}{2E}$

$$\delta = \int_0^l \frac{f(x)\,dx}{AE}$$

$\delta = \frac{\delta_A l (l/2)}{AE}$ $\delta_A l$ l A, δ $l/2$ P

$\delta_A x + P = f(x)$ $\delta_A x$ x P

$$= \frac{1}{AE}\int_0^l (\delta_A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\delta_A x^2}{2} + Px\Big|_0^l\right)$$

$$= \frac{\delta l^2}{2E} + \frac{Pl}{AE}$$

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حل تمرین مقاومت ۱

فصل ۱ : 37 _ 33 _ 65 _ 43 _ 18

sec 1 → (2-15), (2-19), (2-21), (2-23), (2-30)

sec 2 → (2-37), 2-41, 2-43, (2-45), (2-46)

sec 3 → 2-49, 2-55, (2-60)

sec 4 → 2-73, 2-74, 2-83, 2-84

sec 5 → (1-9), (1-15), (1-20), (1-28)

sec 6 → 2-66, 2-75, 2-76

sec 7 → (3-11), (3-21), 3-48, 3-50, 3-61, 3-63

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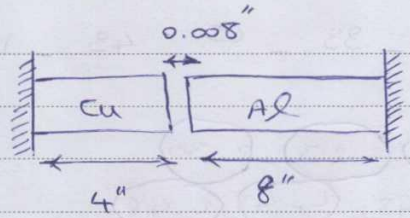


$T_1 = 60\ ^\circ F$

$d = 1.25"$

$\alpha_{Al} = 13 \times 10^{-6}\ 1/F$

$E_{Al} = 10 \times 10^3\ ksi$

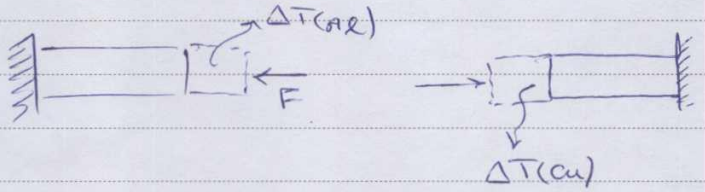
$\alpha_{Cu} = 9.4 \times 10^{-6}\ 1/F$

$E_{Cu} = 18 \times 10^3\ ksi$

① $T_2 = ?$ برای P شود gap بسته

② $T_2 = 200^\circ F \Rightarrow P = ?$

③ $T_2 = 300^\circ F \Rightarrow L_{(Al)} = ?$



① $\delta_T = 0.008 = \alpha_{Al} \underbrace{\Delta T}_{(T_2 - 60)} L_{Al} + \alpha_{Cu} \Delta T L_{Cu} \Rightarrow T_2 = 116\ ^\circ F$

② $0.008 = (\delta T)_{Cu} - (\delta F)_{Cu} + (\delta T)_{Al} - \underbrace{(\delta F)_{Al}}_{(\delta F)_{Cu}}$

$$= 9.4 \times 10^{-6}(200-60)4 - \frac{F(4)}{\frac{\pi}{4}(1.25)^2 18 \times 10^3} + 13 \times 10^{-6}(200-60)8 - \frac{F(8)}{\frac{\pi}{4}(1.25)^2 (10 \times 10^3)}$$

$\Rightarrow F = 14.195\ kip$

$$\sigma_{Al} = \sigma_{Cu} = \frac{14.195}{\frac{\pi}{4}(1.25)^2} = 11.6\ ksi$$

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③ $T_L=300$ عملیات مشابه ② برای $\Rightarrow F = 31.194$ kip

$$\delta_{Al} = (\delta_T)_{Al} - (\delta_F)_{Al} = 0.00426''$$

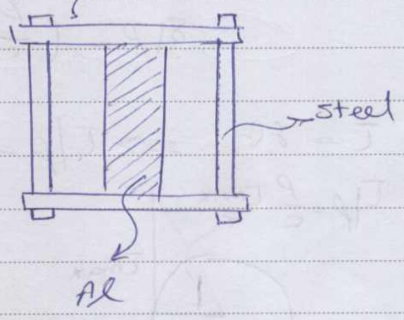
$$L_{Al} = 8 + 0.00426 = 8.00426''$$

this figure makes no sense.

مثال:

$$\begin{cases} d_{Al} = 50 \text{ mm} \\ d_{st} = 10 \text{ mm} \end{cases}$$

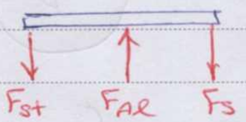
$$\begin{cases} l_{Al} = 100 \text{ mm} \\ l_{st} = 150 \text{ mm} \end{cases}$$



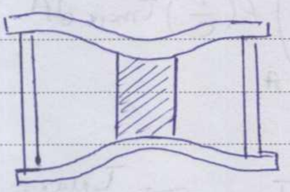
$$\begin{cases} \alpha_{st} = 17 \times 10^{-6}\ 1/^\circ C \\ \alpha_{Al} = 26 \times 10^{-6}\ 1/^\circ C \end{cases}$$

$$\begin{cases} E_{st} = 193 \times 10^9\ Pa \\ E_{Al} = 44.7 \times 10^9\ Pa \end{cases}$$

$\Delta T = 110$



$$\Rightarrow \begin{cases} 2F_{st} = F_{Al} \\ \delta_{Al} = \delta_{st} \end{cases}$$



$$(\delta_T + \delta_F)_{st} = (\delta_T - \delta_F)_{Al}$$

$$17 \times 10^{-6} \times 0.15 \times 110 + \frac{F(0.15)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2} = 26 \times 10^{-6} \times 0.1 \times 110 - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2}$$

$$\Rightarrow F = 904\ N$$

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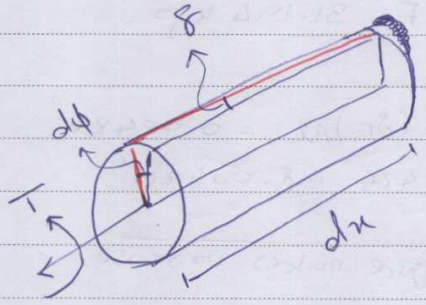
77-60-21-17 (2)

: 3 شعاع



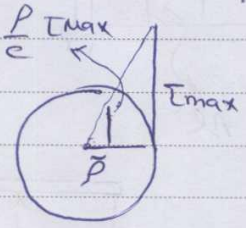
$$c\,d\phi = \gamma\,dx$$

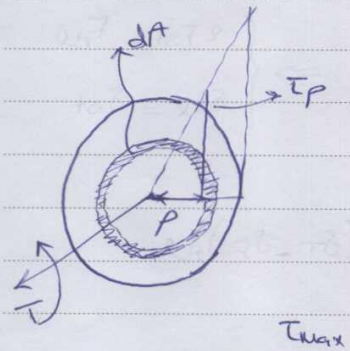
$$\Rightarrow \gamma = \frac{c\,d\phi}{dx}$$

$$\gamma|_\rho = \left(\frac{\rho}{c}\right)\gamma_{max}$$

$$\tau = \gamma G \Rightarrow \tau|_\rho = \frac{\rho}{c}\,\tau_{max}$$

$$\tau|_\rho = \frac{\rho}{c}\,\tau_{max}$$





$$T = \int_A \rho(\tau_\rho\,dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{max}\,dA$$

$$= \frac{\tau_{max}}{c}\int_A \rho^2\,dA = \frac{\tau_{max}}{c}J \longrightarrow \tau_{max} = \frac{Tc}{J}$$

$$\tau_\rho = \frac{T\rho}{J}$$

$$J = I_x + I_y = \int \underbrace{(x^2+y^2)}_{\rho^2}\,dA$$

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برای سطح مقطع دایره‌ای $\bar{J} = \int_A \rho^2 dA = \int_0^c \rho^2 (2\pi\rho d\rho) = 2\pi \int_0^c \rho^3 d\rho$

$$\Rightarrow \boxed{\bar{J} = \frac{\pi}{2} c^4}$$

برای دایره توخالی $\bar{J} = 2\pi \int_{R_i}^{R_o} \rho^3 d\rho = \boxed{\frac{\pi}{2}(R_o^4 - R_i^4)}$

$$c\,d\phi = \delta\, dx \Rightarrow \rho\, d\phi = \delta_\rho\, dx$$

$$\Rightarrow d\phi = \frac{\delta_\rho}{\rho} dx$$

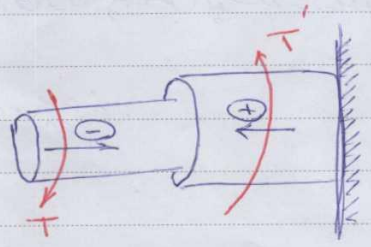
$$\tau = \delta G \Rightarrow \delta = \frac{\tau}{G}$$

$$\Rightarrow \delta_\rho = \frac{\tau\rho}{\bar{J}G}$$

$$\tau_\rho = \frac{T\rho}{\bar{J}}$$

$$\Rightarrow d\phi = \frac{T\rho}{\bar{J}G\rho} dx \Rightarrow \boxed{\phi = \int_0^l \frac{T}{\bar{J}G} dx}$$

$$\Rightarrow \phi = \frac{Tl}{\bar{J}G}$$



$$\phi = \sum_{i=1}^{n} \frac{T_i l_i}{\bar{J}_i G_i}$$

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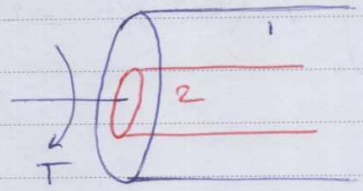
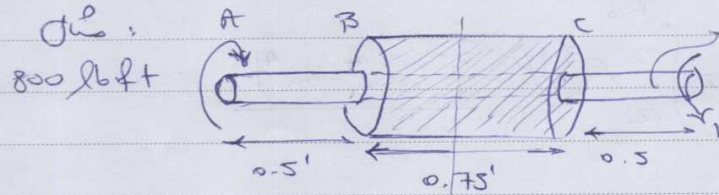


$T = T_1 + T_2$

$\phi_1 = \phi_2$



$800 \times 12 = T_t + T_{sh}$

$\phi_t = \phi_{sh}$

$$\frac{T_t (0.75 \times 12)}{\frac{\pi}{2}(1.5^4 - 1.25^4)G} = \frac{T_{sh}(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$$\begin{cases} T_t = 9376.42 \text{ lb-in} \\ T_{sh} = 223.58 \text{ lb-in} \end{cases}$$

$$\phi_{A/D} = \phi_{A/B} + \phi_{B/C} + \phi_{C/D} = \left[\frac{TL}{GJ}\right]$$

$$= 2 \times \frac{800 \times 12 \times 0.5 \times 12}{\frac{\pi}{2}(0.5)^4 (11 \times 10^6)} + \frac{223.58(0.75)12}{\frac{\pi}{2}(0.5)^4 (11 \times 10^6)^2}$$

برای ϕ_{BC} هم می‌توان چرخش پوسته را در نظر گرفت هم چرخش میله که این فرض

کرده ایم این دو با هم برابرند.

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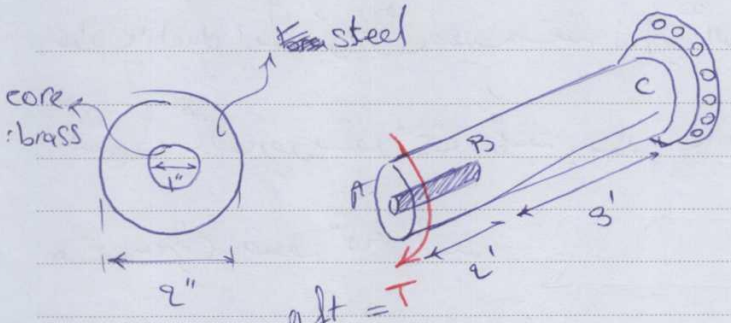
مثال:



$$\tau_{max}|_{brass} = \frac{T_{br}\,0.5}{J_{br}} \qquad T_{br} + T_{st} = 50 \quad (1)$$

$$\phi_{A/B} = \frac{T_{br}(2)(12)}{\frac{\pi}{2}(0.5)^4(5.6)(10^6)} = \frac{T_{st}(2)(12)}{\frac{\pi}{2}(1^4 - 0.5^4)11.5(10^6)} \quad (2)$$

$$\Rightarrow (1),(2) \Rightarrow T_{st} = 48.42$$

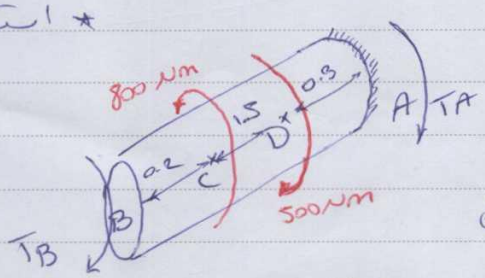
$$T_{br} = 1.57$$

$$\phi = \sum \frac{TL}{JG} = 0.116^\circ$$

τ_{max} برای brasi قطعه در سطح آن اتفاق می‌افتد. برای steel ممکن است در سطح مشترک AB باشد یا BC. AB آلومینیوم و BC آهن است.

* استاتیکی نامعین *

مثال: τ_{max} کدام قسمت؟



$$-T_B + 800 - 500 - T_A = 0$$

$$\phi_{B/A} = 0$$

$$T_A = -345 \text{ Nm}$$

$$T_B = 645 \text{ Nm}$$

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مواد ductile از تنش برشی ضعیف هستند بنابراین اگر بخواهیم ماده ductile نیروی

کششی وارد شود ماده در صفحه 45° می‌شکند. مواد brittle از تنش محوری عمود ضعیف

هستند بنابراین در صفحه 90° می‌شکنند

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$dW = T\,d\theta$ (16) work = torque × angle of twist

$P = \frac{dW}{dt} = T\frac{d\theta}{dt} = T\omega$ power = $\frac{work}{time}$

(ω: سرعت زاویه‌ای)

$P = T\omega$ → rad/sec

Watt ← Nm

SI: 1 watt = 1 Nm/sec

British: $P \equiv$ ft.lb/sec

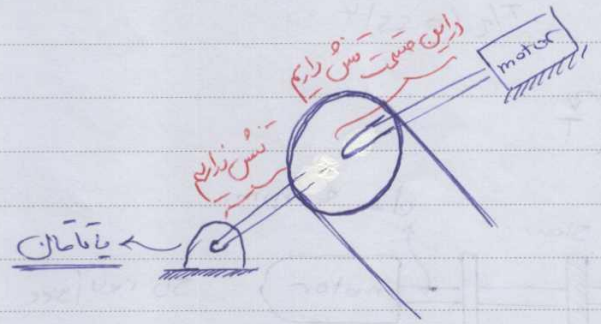
$T \equiv$ ft.lb

1 hp = 550 ft.lb/sec

مثال: یک شفت توپر برای انتقال قدرت 5 hp با سرعت $\omega = 175$ rpm

rpm ← round per minute

$\tau_{all} = 14.5$ ksi . مقاومت شفت چقدر است.



$P = T\omega$ $\omega = 175\,(2\pi)\frac{1}{60} = 18.33$ rad/sec

$P = 5(550) = 2750$ ft lb/sec

$T = \frac{P}{\omega} = 150.1$ ft lb

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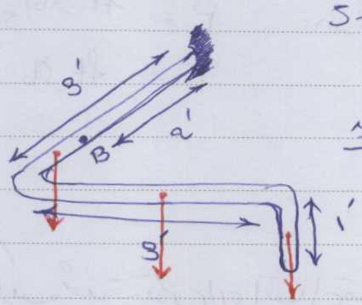
$$\tau_{all} = \frac{Tc}{J} \Rightarrow \frac{J}{c} = \frac{T}{\tau_{all}} \Rightarrow c = \left(\frac{2T}{\pi \tau_{all}}\right)^{1/3} \qquad \left(J = \frac{\pi}{2}c^4\right)$$

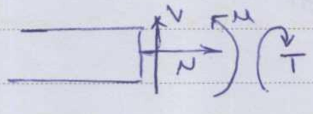
$$\Rightarrow c = \left(\frac{2 \times 150.1 \times 12}{\pi (14500)}\right)^{1/3} \Rightarrow c = 0.429'' \Rightarrow d = 0.858''$$

مثال: میله سنی زیر قطر "0.5 وزن 5 lb/ft را دارا است. ماکزیمم تنش برشی را در مقطع B بدست آورید

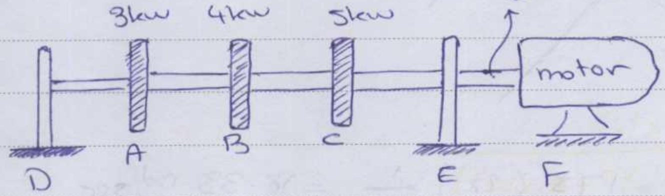


$$T = \Sigma M = 15(1.5) + 5(3) = 37.5 \text{ lb.ft} = 450 \text{ lb in}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{450(0.25)}{\pi/2\,(0.25)^4} = 18.3 \text{ ksi}$$



مثال:



50 rev/sec

$$\omega = 50(2\pi) = 100\pi \text{ rad/sec}$$

$$T_{CF} = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.2 \text{ Nm}$$

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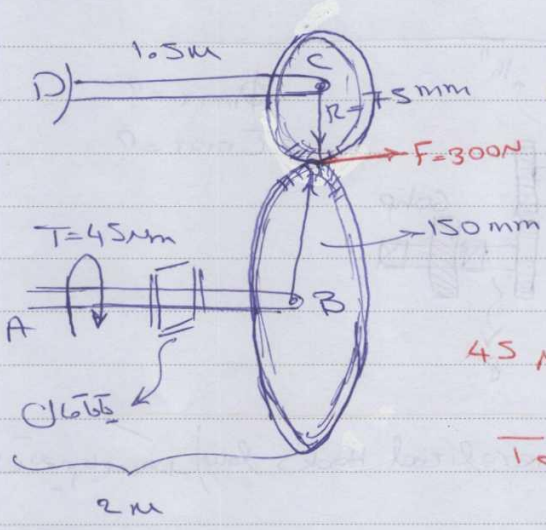
$\tau_{AB} = \frac{3(10^9)}{100\pi} =$

$\tau_{BC} = \frac{7(10^3)}{100\pi} =$

$\tau_{max} = \tau_{CF} = \frac{Tc}{J}$

$\phi_{DF} = \{ \phi = \phi_{AB} + \phi_{BC} + \phi_{CF}$

$\phi_{AB} = \frac{T_{AB} L}{JG}$



$G = 80\ GPa$: Given

$d = 20\ mm$

ϕ: twist A = ?

$45\ Nm = F \times 0.15\ m \Rightarrow F = 300N$

$T_{CD} = 300 \times 0.075 = 22.5\ Nm$

$\phi_{C/D} = \frac{22.5 \times 1.5}{\frac{\pi}{2}(0.01)^4 (80 \times 10^9)} = 0.0269\ rad$

$\phi_B R_B = \phi_C R_C$

$\phi_B (1.5) = 0.0269 \times 0.75 \rightarrow \phi_{B/D} = 0.0134\ rad$

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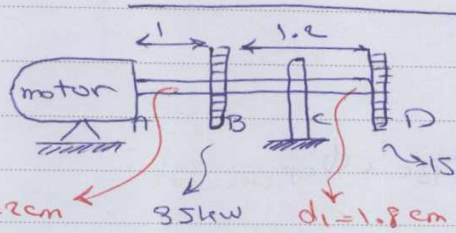
$$\phi_{A/B} = \frac{45 \times 2}{\frac{\pi}{2}(0.01)^4 (80 \times 10^9)} = 0.0716 \text{ rad}$$

$$\phi_{A/D} = \phi_{A/B} + \phi_{B/D} = 0.085 \text{ rad}$$

مثال:



P = 50 kw

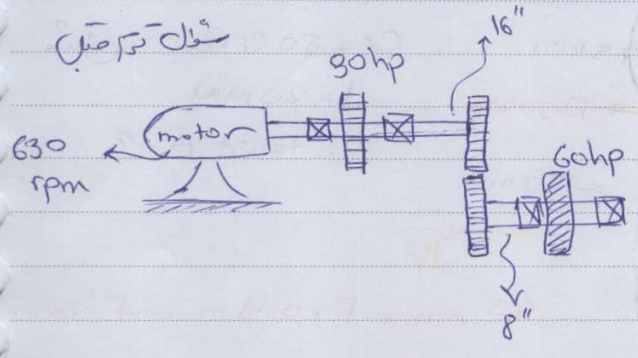
ω = 62.8 rad/sec

τall

d = ?

$35 + 15 \leq 50$ ✓ مقدار قابل قبول است!

مثال: تمرین



$\phi_{max} = ?$

$\tau_{max} = ?$

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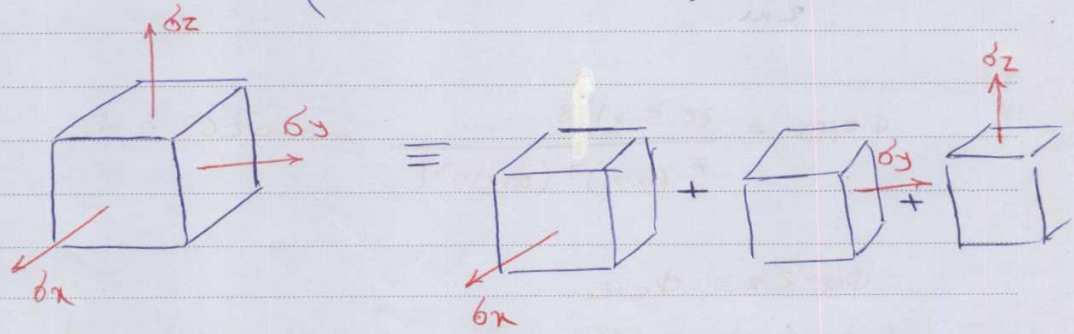
* (generalized Hook's law) قانون تعمیم یافته هوک *



$$\varepsilon_x = \varepsilon'_x + \varepsilon''_x + \varepsilon'''_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

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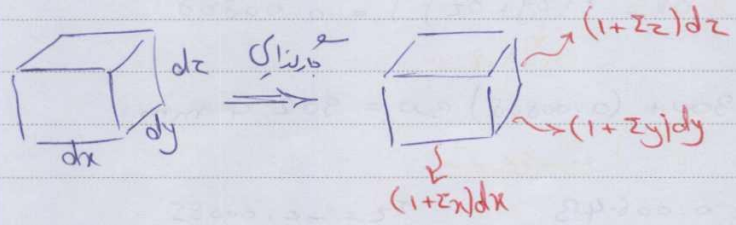
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y+\sigma_z)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x+\sigma_z)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x+\sigma_y)\right)$$



تغییر حجم $\delta V = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)dx\,dy\,dz - dx\,dy\,dz$

$$\Rightarrow \delta V \simeq (\varepsilon_x+\varepsilon_y+\varepsilon_z)\underbrace{dx\,dy\,dz}_{\Delta V}$$

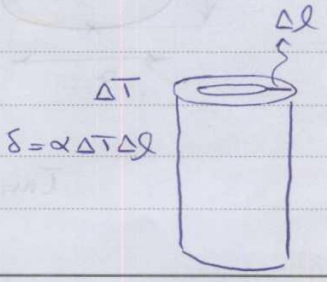
$$e = \frac{\delta V}{\Delta V} = \varepsilon_x+\varepsilon_y+\varepsilon_z$$

کرنش حجمی به چه معنی است.

با استفاده از قوانین تعمیم یافته هوک مقدار کرنش حجمی بصورت زیر می‌توان نوشت:

$$e = \frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z)$$

$$G = \frac{E}{(1+\nu)2}$$



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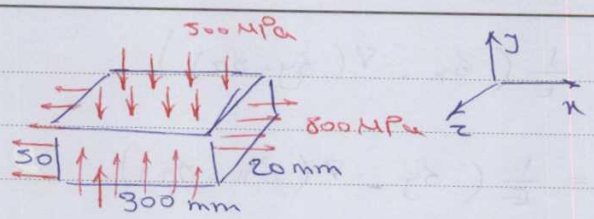
مثال:

$E = 120\ GPa$

$\nu = 0.34$



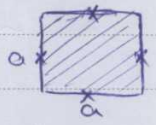
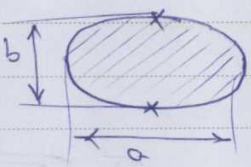
$\sigma_x = 800 MPa$, $\sigma_y = -500 MPa$, $\sigma_z = 0$, $\tau = 0$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) = 0.00808$$

طول جدید $= 300 + (0.00808)\,300 = 302.4\ mm$

$\varepsilon_y = -0.00643$ $\quad$ $\varepsilon_z = -0.00085$

عرض جدید $= 19.98\ mm$ $\quad$ ارتفاع جدید $= 49.68\ mm$

	τ_{max}	ϕ	τ_{max} و ϕ برای آلومینیوم
	$\frac{4.81T}{a^3}$	$\frac{7.1Tl}{a^4G}$	
	$\frac{20T}{a^3}$	$\frac{46Tl}{a^4G}$	
	$\frac{2T}{\pi ab^2}$	$\frac{(a^2+b^2)Tl}{\pi(a^3b^3)G}$	

$$\tau_{max} = \frac{Tc}{J} \qquad \phi = \frac{Tl}{JG} \quad ; \quad J = \frac{1}{2}\pi c^4$$

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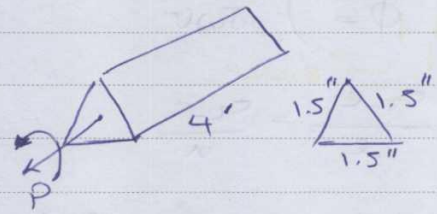
مثال:



$\tau_{all} = 8\,ksi$

$\phi_{All} = 0.02\ rad$

$T_{max} = ?$

$\tau_{all} = \frac{20T}{a^3} \Rightarrow 8\times10^3 = \frac{20T}{(1.5)^3}$

$\Rightarrow T = 1350\ lb\,in$

$\phi = \frac{46Tl}{a^4 G} = \frac{46T(4\times12)}{(1.5)^4(3.7\times10^6)} \Rightarrow T = 170\ lb\text{-}in$

$\Rightarrow T_{max} = 170$

$\rho V_c = \rho V_t \Rightarrow V_c = V_t \Rightarrow \ell A_c = \ell A_t \Rightarrow A_c = A_t$

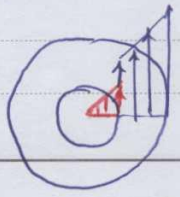
$\Rightarrow \pi R^2 = \frac{1}{2}(1.5)(1.5)\sin 60 \Rightarrow R = 0.577"$

$\tau_{all} = \frac{TR}{J} \Rightarrow 8\times10^3 = \frac{T(0.577)}{\frac{\pi}{2}(0.577)^4} \Rightarrow T = 2170\ lb\,in$

$\phi_{All} = \frac{Tl}{JG} \Rightarrow T = 233\ lb\,in$

$T = T_{st} + T_{Br}$

$\phi_{st} = \phi_{Br}$ $\Rightarrow$ T_{Br} ✓ , T_{st} ✓



تنش نامشخص داریم اما تغییر شکل یکسانی داریم

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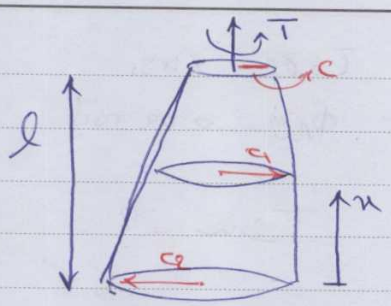

$$\phi = \int_0^l \frac{T\,dx}{J(x)G}$$

$$\frac{c_2 - c_1}{l} = \frac{c_2 - c}{x}$$

$$\Rightarrow c = c_2 - x\,\frac{c_2 - c_1}{l}$$

$$J(x) = \frac{\pi}{2}\left(c_2 - x\,\frac{c_2 - c_1}{l}\right)^4$$

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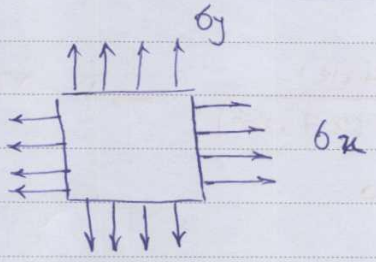

$$\sigma_x = E\left(\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2}\right) \qquad \sigma_y = E\left(\frac{\nu\varepsilon_x + \varepsilon_y}{1-\nu^2}\right)$$

$$\sigma_x = A\varepsilon_x + B\varepsilon_y$$

$$\sigma_y = C\varepsilon_x + D\varepsilon_y$$

$$\sigma_y = 0 \Rightarrow \varepsilon_x = \frac{\sigma_x}{E},\quad \varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\sigma_x = 0 \Rightarrow \varepsilon_y = \frac{\sigma_y}{E},\quad \varepsilon_x = -\nu\frac{\sigma_y}{E}$$

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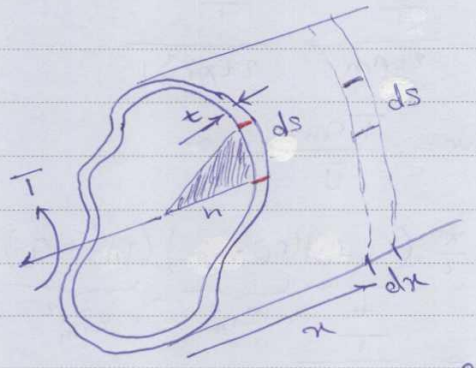
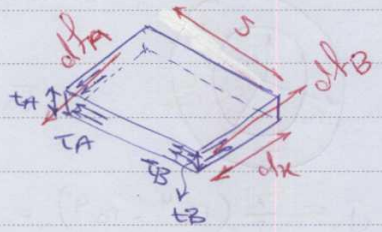




$$\tau_A (t_A dx) = \tau_B (t_B dx)$$

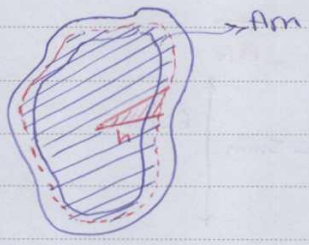
$$\Rightarrow \tau_A t_A = \tau_B t_B$$

جریان برشی $= q = \tau_{av} t = const$

$$df = \tau_{ave}\, dA = \tau_{ave}\,(t ds)$$

$$dT = h(df) = h(\tau_{ave}\, t\, ds) \Rightarrow T = \oint h \underbrace{\tau_{ave} t}_{q=const} ds$$

$$\Rightarrow T = \tau_{ave} t \oint h ds = 2\tau_{ave} t \oint dA_m = 2\tau_{ave} t A_m$$

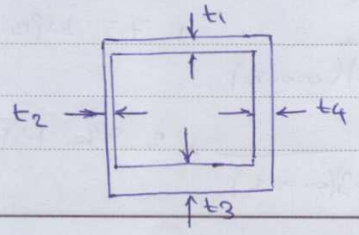


$$dA_m = \tfrac{1}{2} h ds$$

$$\Rightarrow \tau_{ave} = \frac{T}{2tA_m}$$

$$q = \frac{T}{2A_m}$$

$$\text{ثابت } D \Rightarrow \phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$



برای ϕ ، دهرکب از 4 قسمت t ثابت
پس 4 انتگرال جداگانه می‌گیریم

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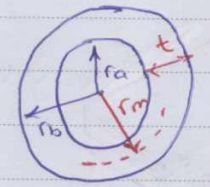


$$\tau_{ave} = \frac{T}{2tA_m} = \frac{T}{2t\pi r_m^2}$$

$$\Rightarrow \tau_{ave} = \frac{Tr_m}{J} \quad \text{ee}$$

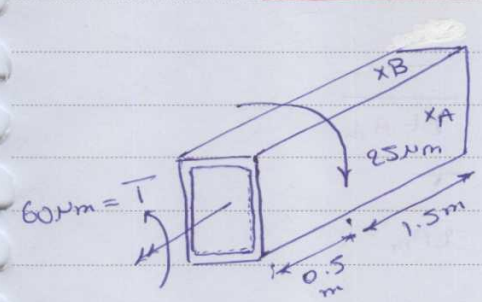
$$J = \frac{\pi}{2}(r_b^4 - r_a^4) = \frac{\pi}{2}\underbrace{(r_b - r_a)}_{t}\underbrace{(r_b + r_a)}_{2r_m}\underbrace{(r_a^2 + r_b)^2}_{2r_m^2} = 2\pi r_m^3 t$$

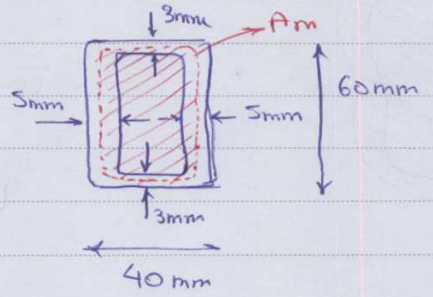
$$\text{ee} \Rightarrow \tau_{ave} = \frac{Tr_m}{2\pi r_m^3 t} = \frac{T}{2t\pi r_m^2}$$

$$\phi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{t} = \frac{TL}{4(\pi r_m^2)^2 G t}\underbrace{\oint ds}_{2\pi r_m}$$

ثابت

$$\Rightarrow \phi = \frac{TL}{\underbrace{(2\pi r_m^3 t)}_{J} G} = \frac{TL}{JG}$$





τ_A, τ_B ?

total ϕ

$$\tau_A = \frac{T}{2t_A A_m} \qquad A_m = (0.057)(0.035) = 0.002\ m^2$$

$$\tau_A = \frac{35}{2(0.005)(0.002)} = 1.75\ MPa$$

$$\tau_B = \frac{35}{2(0.003)(0.002)} = 2.92\ MPa$$

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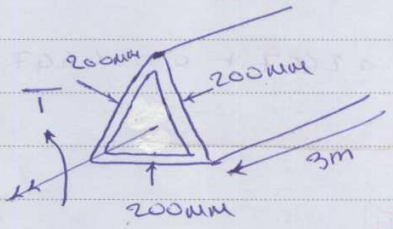
$$\phi = \sum \frac{TL}{4Am^2G} \oint \frac{ds}{t} = \frac{60 \times (0.5)}{4(0.002)^2 \times 35(10^9)} \left[2\frac{57}{5} + 2\frac{35}{3} \right]$$

$$+ \frac{35(1.5)}{4(0.002)^2 \times 35(10^9)} \left[2\frac{57}{5} + 2\frac{35}{3} \right] = 6.29\,(10^{-3})\ \text{rad}$$



$T_{all} = 90\,MPa$

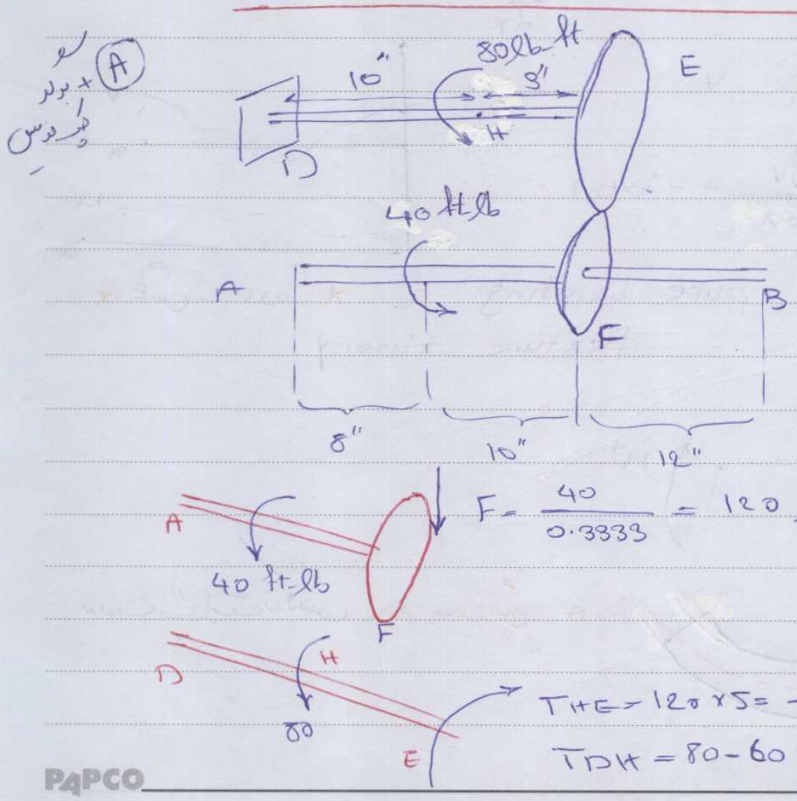
made of 5mm plate

$\phi_{max} = 2 \times 10^{-3}$ rad

$T_{max} = ?$

$T_1 = 15.6\,kN.m$

$T_2 = 500\,N.m$ } $T_{max} = 500\,N.m$



$\phi_{A)\,total} = ?$

$F = \frac{40}{0.3333} = 120\ lb$

$T_{HE} = 120 \times 5 = -60$

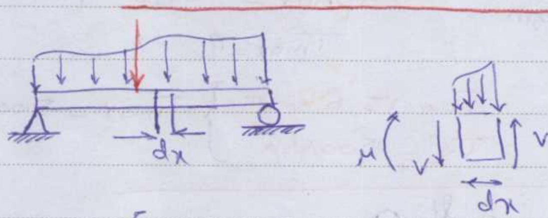
$T_{DH} = 80 - 60 = 20$

$$\phi_E = \frac{-60(12\times30)+20(12\times10)}{\frac{\pi}{2}(0.5)^4(11\times10^6)} = 0.01778 \text{ rad "cw"}$$

$$\phi_F = \frac{6}{4}\phi_E = 0.02667 \text{ (ccw)}$$

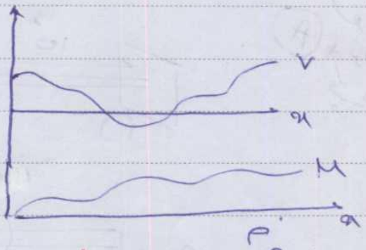
$$\phi_{A/F} = \frac{-40(12)(10)}{\frac{\pi}{2}(0.5)^4(11\times10^6)} = -0.004445 \text{ (ccw)}$$

$$\phi_A\big)_{total} = \phi_F + \phi_{A/F} = 0.02667 + 0.004447 = \ldots$$



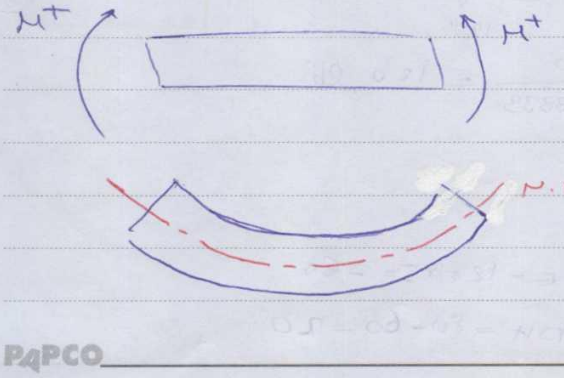
معادلات تعادل برای المان ⟹

$$\begin{cases} \dfrac{dM}{dx} = V \\[2ex] \dfrac{dV}{dx} = -W(x) \end{cases}$$



pure bending ★ خمش ساده ★

flexture theory



نقاطی که تنش ندارند ← محور خنثی N.A

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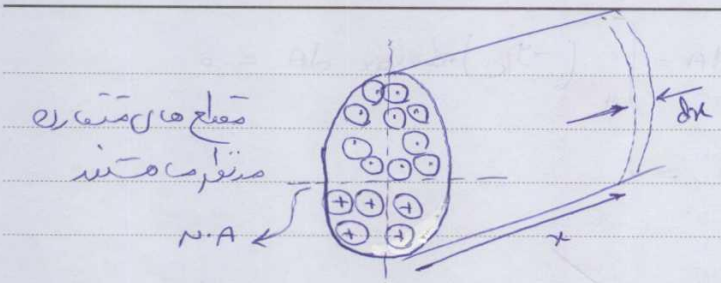
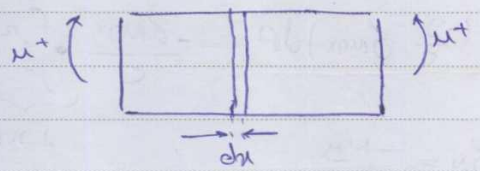
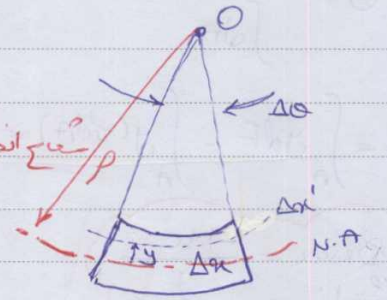


محور تقارن



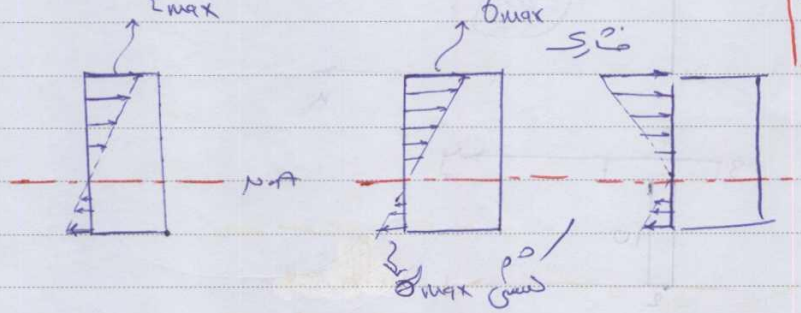


$$\varepsilon_y = \lim_{\Delta x \to 0} \frac{\Delta x' - \Delta x}{\Delta x}$$

$$\Delta x = \rho \Delta\theta \quad , \quad \Delta x' = (\rho - y)\Delta\theta$$

$$\varepsilon_y = \lim_{\Delta x \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} \longrightarrow \boxed{\varepsilon_y = \frac{-y}{\rho}}$$

$$\boxed{\varepsilon_y = y/c\ \varepsilon_{max}}$$



$$\sigma = E\varepsilon$$

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$$\sum F = 0 \longrightarrow \int_A \sigma\, dA = \int_A (-y_k)\,\sigma_{max}\, dA = 0$$

$$\Rightarrow \frac{\sigma_{max}}{c} \int_A y\, dA = 0$$

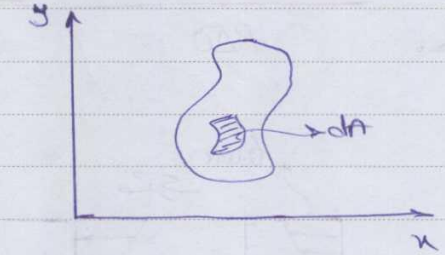
$$\bar{y} = \frac{\int y\, dA}{\int dA}$$ حداقل فاصله از مرکز سطح (centroid) سطح عبور می‌کند

$$M = \int_A y\, dF = \int_A y(\sigma\, dA) = \int_A y\left(\frac{-y}{c}\sigma_{max}\right) dA = \frac{-\sigma_{max}}{c} \cdot \underbrace{\int y^2\, dA}_{I_{xx}}$$

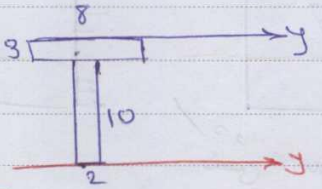
$$\sigma_{max} = \frac{-Mc}{I}$$ که نسبت مستقیم با لنگر خمشی دارد

$$\sigma_y = \frac{-My}{I}$$

$$\bar{y} = \frac{\int y\, dA}{\int dA} = \frac{\sum \bar{y} A}{\sum A}, \quad I_{xx} = \int y^2\, dA$$



مثال:



$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{(2\times10)(5) + (3\times8)(11.5)}{2\times10 + 3\times8} = 8.55$$ از بالا

$$\bar{y} = \frac{(3\times8)(-1.5) + (10\times2)(-8)}{3\times8 + 10\times2} = -4.45$$ از پایین

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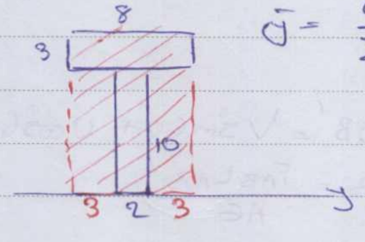
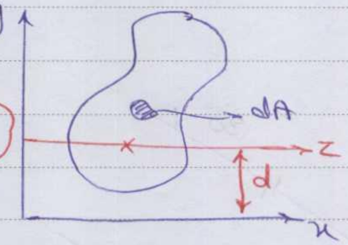


$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{(13\times 8)(6.5) - 2(10\times 3)(5)}{13\times 8 - 2\times(10\times 3)}$$

$$= 8.55$$

محور نزدیک‌تر را مد نظر قرار دهید

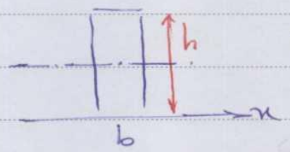
$$I_x = I_z + A d^2$$



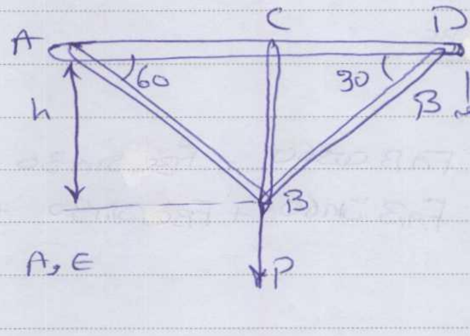
$$I_x = \int_A y^2 \, dA$$

$$I_y = \int_A x^2 \, dA$$

$$J = \int_A r^2 \, dA = \int_A (x^2 + y^2)\, dA = I_x + I_y$$



$$I_{N.A} = \frac{1}{12} b h^3 \rightarrow I_x = \frac{1}{12} b h^3 + (bh)(h/2)^2$$
$$= \frac{1}{3} b h^3$$



جابجایی افقی و عمودی نقطه B

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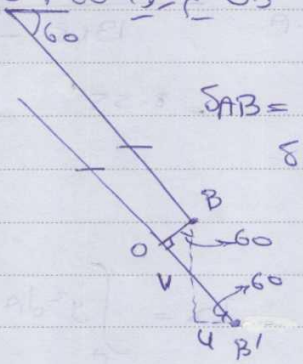
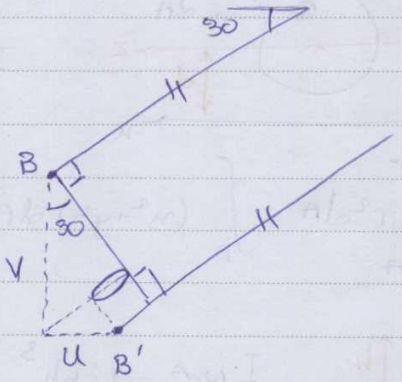
فرض کنیم زاویه 60 و 60 باقی بماند



$$\delta_{AB} = OB' = V\sin 60 + U\cos 60$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{AE}$$



$$\delta_{BD} = V\sin 30 - U\cos 30 = \frac{F_{BD} L_{BD}}{AE}$$

$$\delta_{BC} = V = \frac{F_{BC} L_{BC}}{AE}$$

$$\sum F_x = 0 \Rightarrow F_{AB}\cos 60 = F_{BD}\sin 30$$

$$\sum F_y = 0 \Rightarrow F_{AB}\sin 60 + F_{BD}\sin 30 + F_{BC} = P$$

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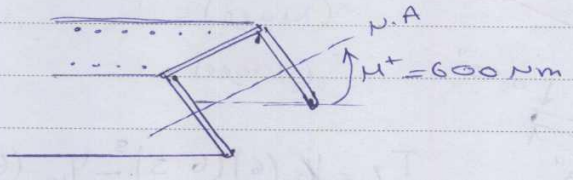
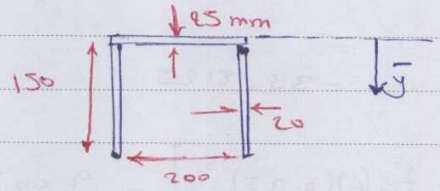
مثال: تنش نرمال حداکثر و حداقل را محاسبه کنید





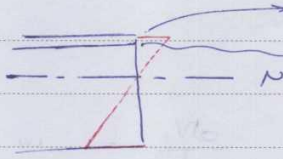
$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(0.24 \times 0.025)(0.0125) + 2(0.15 \times 0.02)(0.025 + 0.075)}{(0.24 \times 0.025) + 2(0.15 \times 0.02)}$$

$$= 0.05625\ m$$

$$I = \sum\left(\frac{bh^3}{12} + Ad^2\right) = \frac{1}{12}(0.24)(0.025)^3 + (0.24)(0.025)(0.05625 - 0.0125)^2$$

$$+ 2 \times \frac{1}{12}(0.02)(0.15)^3 + 2(0.15)(0.02)(0.04375)^2$$

$$= 34.53 \times 10^{-6}\ m^4$$



$$\sigma_{max} = \frac{600(0.05625)}{34.53 \times 10^{-6}} = 0.977\ MPa$$

$$\sigma_{min} = \frac{600(0.175 - 0.05625)}{34.53 \times 10^{-6}} = 0.593$$

$$F = \frac{(0.977 + 0.593)}{2}(0.025 \times 0.24) = 4.56\ kN$$

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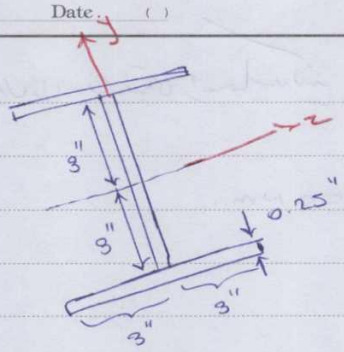


مثال

$\sigma_{all} = 24$ ksi

$(M_{all})_z$

$(M_{all})_y$

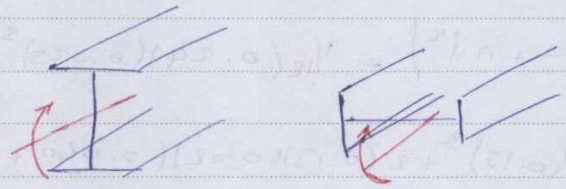
$$I_z = \frac{1}{12}(6)(6.5)^3 - \frac{1}{12}(6-0.25)6^3$$

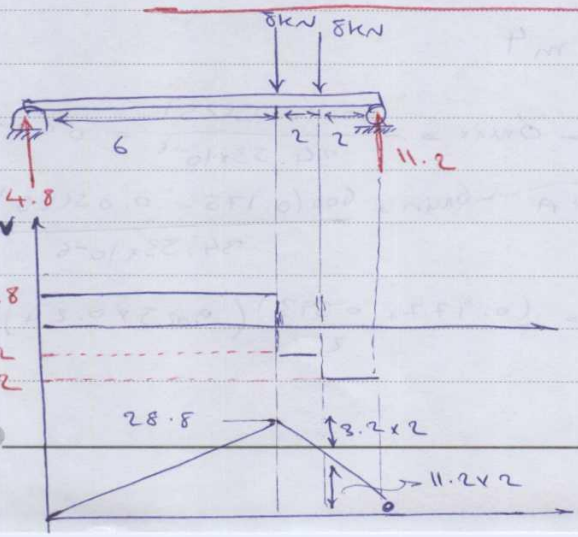
$$= 33.8125$$

$$I_y = 2\left(\frac{1}{12}(0.25)(6)^3 + \frac{1}{12}(6)(0.25)^3\right) = 9.0078$$

$$(M_{all})_z = \frac{\sigma_{all} I_z}{c} = \frac{24(33.8125)}{3.25} = 249.7 \text{ kip-in}$$

$$(M_{all})_y = \frac{\sigma_{all} I_y}{c} = \frac{24(9.0078)}{3} = 72.0625 \text{ kip-in}$$





$$\frac{dV}{dx} = -w$$

$$\frac{dM}{dx} = V$$

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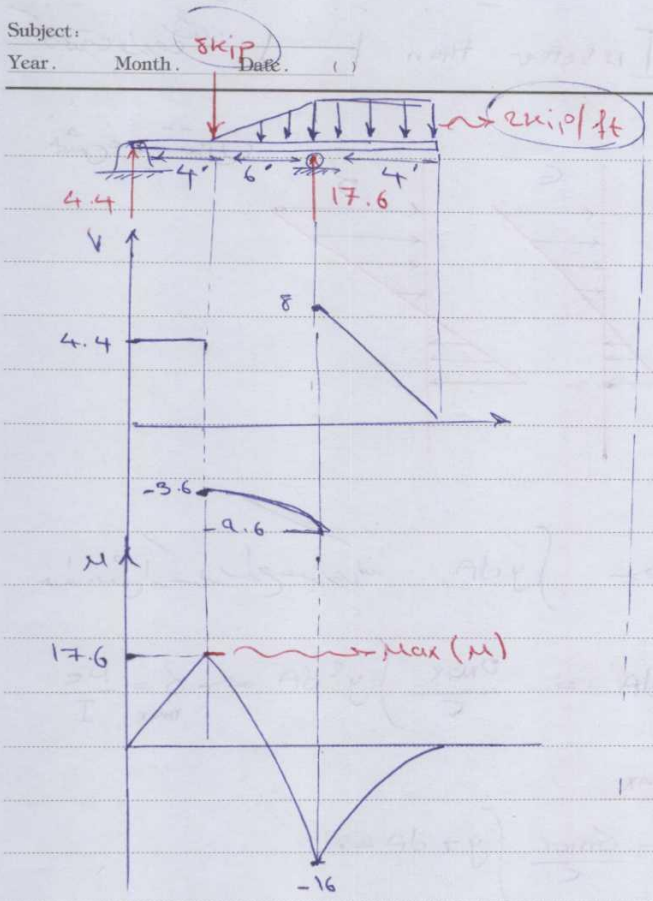

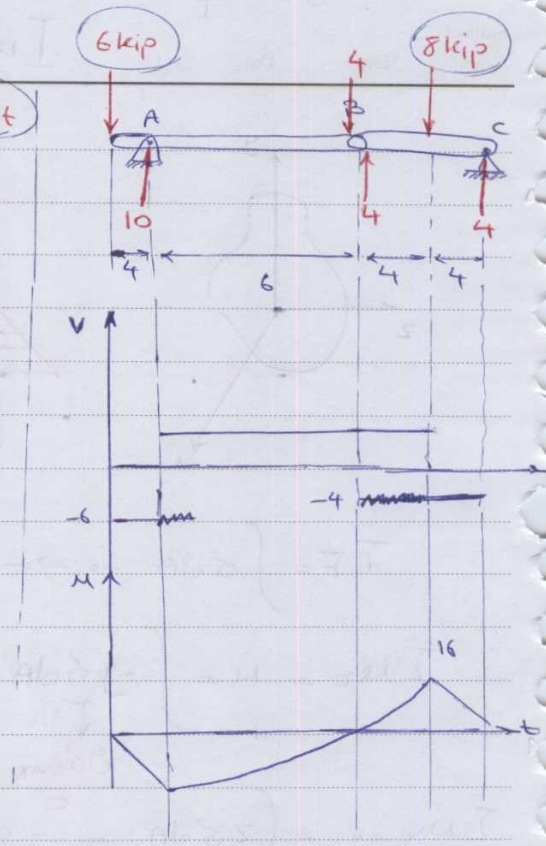

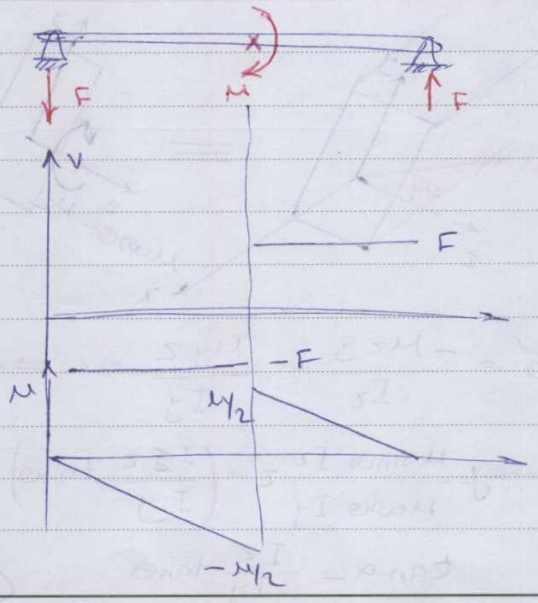

P4PCO

$\sigma = \frac{My}{I}$ ← باید سعی کنیم که I بیشتر باشد تا تنش کمتر وارد شود.

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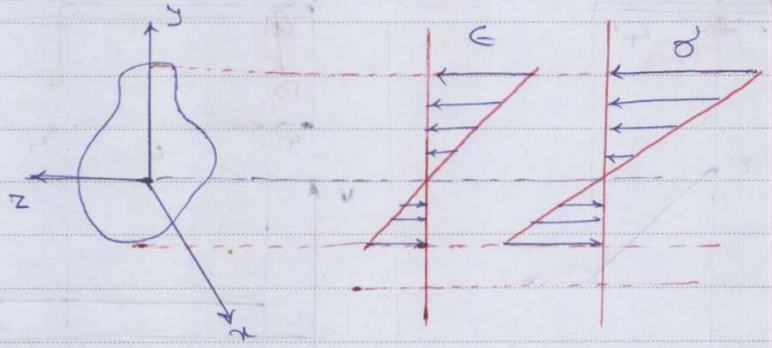
I is better than ⊢

تنش غیر متقارن



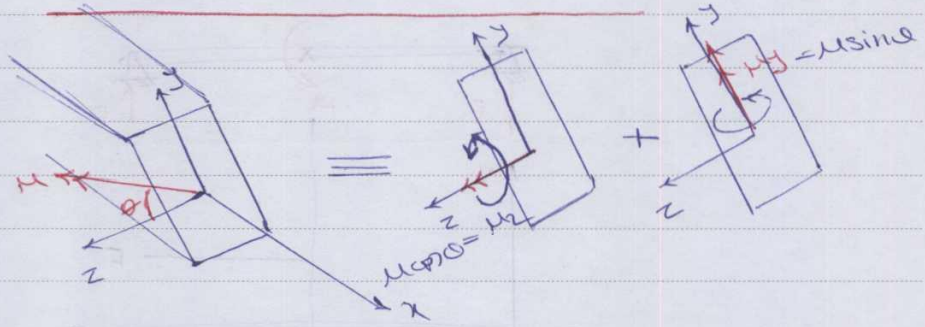
$\sum F = \int \sigma dA = 0 \Rightarrow \int y dA$ نقاطی از مقطع که سطح خنثی از مقطع عبور می‌کند

$\sum M_z = M = \int -y\sigma dA = \frac{\sigma_{max}}{c} \int y^2 dA \Rightarrow \sigma_{max} = \frac{Mc}{I}$

$\frac{\sigma \sigma_{max}}{c}$

$\sum M_y = 0 = \int z\sigma dA = -\frac{\sigma_{max}}{c} \int yz dA = 0$

مثال:



$\sigma = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} = 0 \Rightarrow y = \frac{M_y I_z}{M_z I_y} z$

$\Rightarrow y = \frac{M \sin\theta I_z}{M \cos\theta I_y} z = \left(\frac{I_z}{I_y} \tan\theta\right) z$

$\tan\alpha = \frac{I_z}{I_y} \tan\theta$ شیب خط خنثی از مقطع

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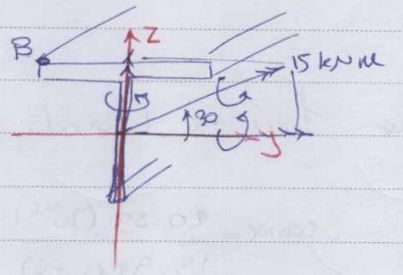
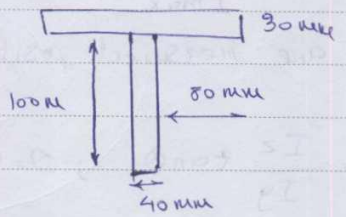
حل:





$M_y = 15\cos 30 = 12.99\ kNm$

$M_z = 15\sin 30 = 7.5\ kNm$

$$\bar{z} = \frac{\sum \bar{z}A}{\sum A} = \frac{0.05(0.1)(0.04) + 0.115(0.03)(0.2)}{(0.1)(0.04) + (0.03)(0.2)}$$

$\rightarrow \bar{z} = 0.089\ m$ (از پایین)

$$I_z = \frac{1}{12}(0.1)(0.04)^3 + \frac{1}{12}(0.03)(0.1)^3 = 20.53(10^{-6})\ m^4$$

$$I_y = \left[\frac{1}{12}(0.04)(0.1)^3 + (0.1)(0.04)(0.089 - 0.05)^2\right] + \left[\frac{1}{12}(0.2)(0.03)^3 + (0.2)(0.03)(0.115 - 0.089)^2\right]$$

$$= 19.92(10^{-6})\ m^4$$

$$\sigma_B = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{(7.5)(-0.1)}{20.53\times10^{-6}} + \frac{12.99(0.041)}{19.92\times10^{-6}} = 74.8\ MPa$$

$$\sigma_C = \frac{-7.5(0.02)}{20.53(10^{-6})} + \frac{12.99(-0.089)}{19.92(10^{-6})} = -90.3\ MPa$$

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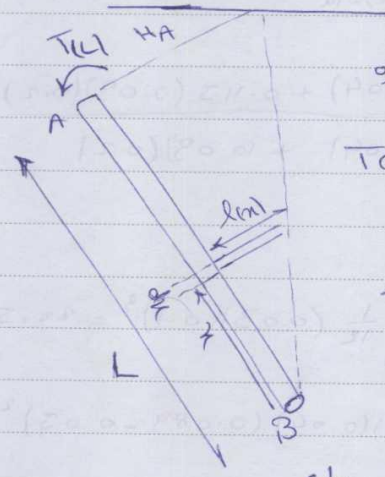
y represents I_{min}

z " I_{max}

θ & α are measured positive from +z towards +y axis.

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta \quad ; \theta = 60 \qquad \tan\alpha = \frac{20.33\,(10^{-6})}{13.92\,(10^{-6})}\tan 60$$

$$\Rightarrow \alpha = 68.6$$



8-4-14

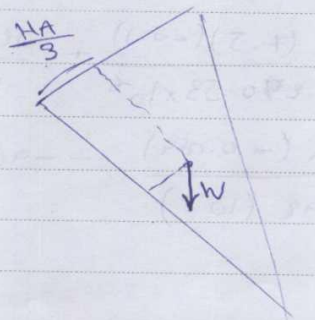
$$T(x) = \underbrace{tpg}_{k}\, l(x)\left(\frac{l(x)}{2}\right)dx$$

$$l(x) = \frac{H_A x}{L}$$

$$W = \int_0^L k\,l(x)\,dx \Rightarrow k = \frac{2W}{L H_A}$$

$$T(L) = \int_0^L \frac{k H_A^2}{2L^2}\,x^2\,dx \Rightarrow T(L) = \frac{H_A W}{3}$$

$$\Rightarrow T_{max} = \dots$$



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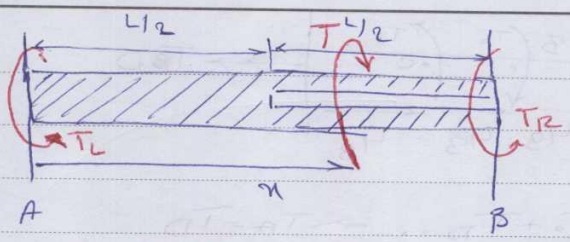


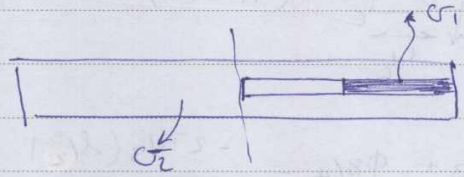
9-8-7

$T_R + T_L = T \xrightarrow{T_R = T_L} T_R = T_L = T/2$

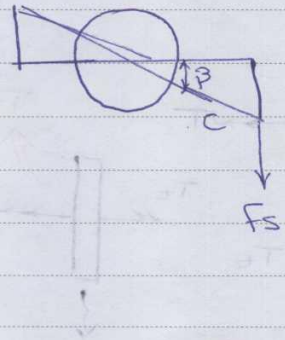
$$\phi_R = \frac{T_R (L-x)}{G J_H} \longrightarrow \phi_R = \phi_L$$

$$\phi_L = \frac{T_L (x - L/2)}{G J_H} + \frac{T_L \, L/2}{G J_N}$$

$$\Rightarrow \frac{T/2\,(L-x)}{J_H} - \frac{T\,(x-L/2)}{J_H} = \frac{\frac{1}{4}L}{J_N}$$



8-8-8



$F_S = k(b - c\beta) \Rightarrow T_S = 2k(b - \beta c)c$

$$\beta = \frac{T_S L}{G I_P}$$

$\Rightarrow \beta \checkmark$

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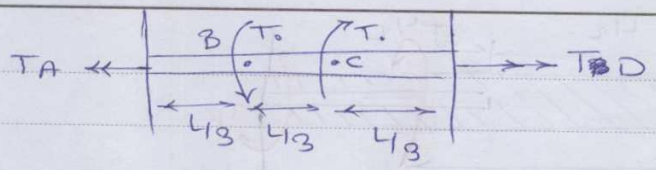


3-22

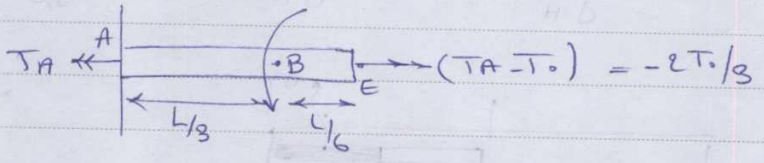
$$-T_A + T_0 - T_0 + T_D = 0 \Rightarrow T_A = T_D$$

$$\phi_{AD} = 0 \Rightarrow \phi_{A/B} + \phi_{B/C} + \phi_{C/D} = 0$$

$$\Rightarrow \frac{T_A(l/3)}{JG} + \frac{(T_A - T_0)l/3}{JG} + \frac{(T_D)l/3}{JG} = 0$$

$$\Rightarrow T_A = T_D = T_0/3$$

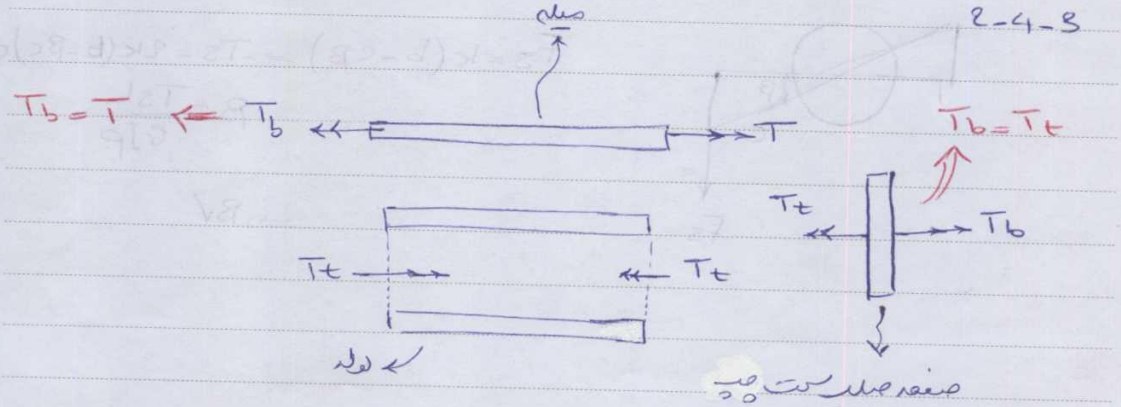
$$\phi_{max} = \phi_{AB} = \frac{\frac{T_0}{3}(l/3)}{JG}$$



$$(T_A - T_0) = -2T_0/3$$

$$\phi_{E/A} = \phi_{E/B} + \phi_{B/A} = \frac{-2T_0/3\,(l/6)}{JG} + \frac{T_0/3\,(l/3)}{JG} = 0$$

2-4-3



P4PCO

$$\tau_{max} = \frac{Tc}{J}$$

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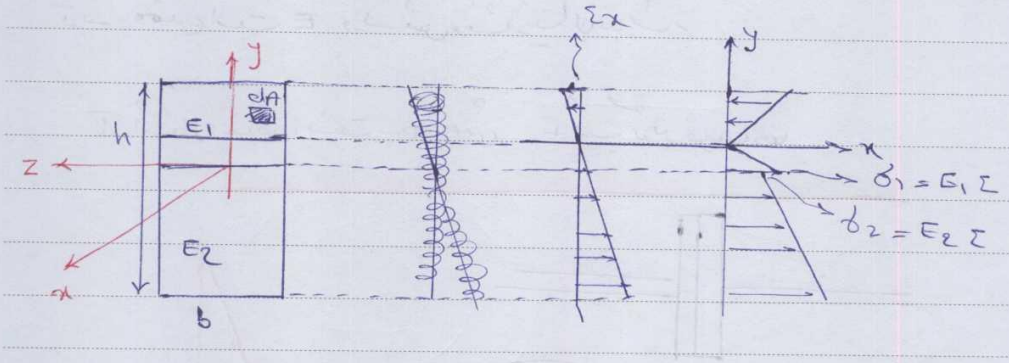


$E_1 > E_2$

$dA = dy\,dz$

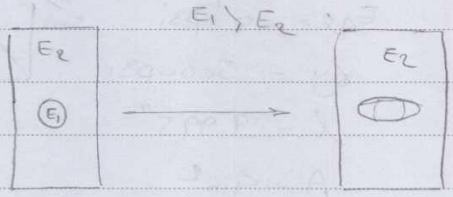
$\sigma_1 dA_1 = (E_1 \varepsilon)\, dy\, dz = dF_2 = \sigma_2 dA_2 = (E_2 \varepsilon) n\, dy\, dz$

($\sigma_1 dA_1 = dF_1$)

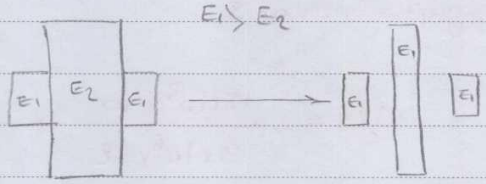
$\Rightarrow n = \frac{E_1}{E_2}$

$y = const \quad dy = const \quad dz_2 = n\, dz_1$

$E_1 > E_2$



$E_1 > E_2$



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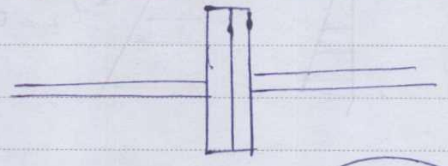
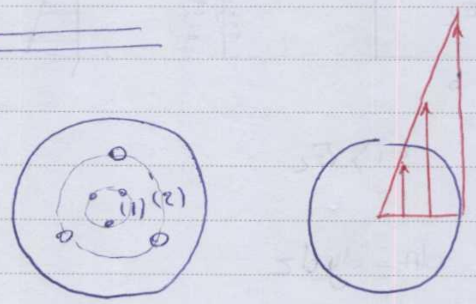
توجه: ممکن است E در کشش و فشار یکسان نباشد

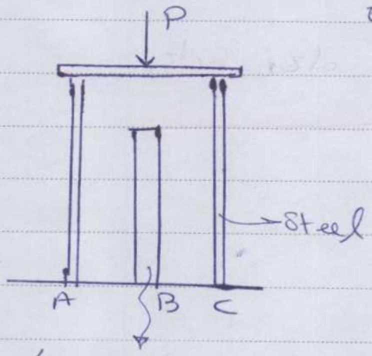
I اشکال خاص نسبت به جوش + پیچ پرچ فرسل ها





$$T = \left(\sum \text{شعاع} * A * \tau_{all}\right) * \text{تعداد پیچ} + \left(\sum \text{شعاع} * A * \tau_{all}\frac{r_1}{r_2}\right) * \text{تعداد پیچ}$$

(2) (1)



$E_s = 3 \times 10^6$ psi

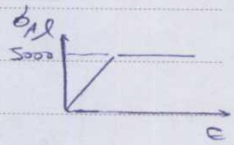
$l = 10''$, $A = 2\,in^2$

$E_{Al} = 10^7$ psi

$\sigma_y = 5000$ psi

$l = 9.995''$

$A = 4\,in^2$



Al الاستیک پلاستیک کامل

(a) gap = 0.005"

(a) P = 30 kips

(b) P = 200 kips

$$\Delta l_s = \frac{15 \times 10^3 \times 10}{3 \times 10^6 \times 2} = 0.025'' > 0.005''$$

gap بسته می‌شود ← نیرو به صفحه وارد می‌شود P

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$$\begin{cases} 2F_S + F_{Al} = P \quad (1) \\ \Delta S = \Delta_{Al} + 0.005 \quad (2) \end{cases} \Rightarrow \frac{F_S(10)}{3\times10^6 \times 2} = \frac{F_{Al}(10)}{4\times10^7} + 0.005$$

$9.995 \simeq$

$(1),(2) \Rightarrow F_S = 5769.23$

$F_{Al} = 18462.54 \qquad \sigma_{Al} = \frac{F_{Al}}{4} = 4615.63 < \sigma_y$

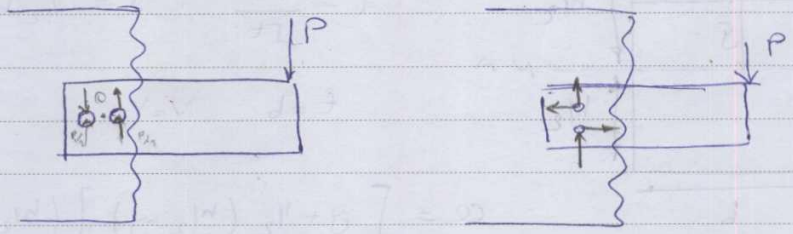
(b) $F_S = 25984.6 \qquad F_{Al} = 19923.78$

$\sigma_{Al} = 9730.794 > \sigma_y \rightarrow$ not accepted

چون آلومینیم بیشتر از σ_y تنش تحمل نمی‌کند

$\sigma_{Al})_{max} = 5000 \Rightarrow F_{Al})_{max}) = 5000 \times 4 = 20000\ lb = R_B$

$F_S = \frac{200000 - 20000}{2} = 90000 = R_A = R_C$



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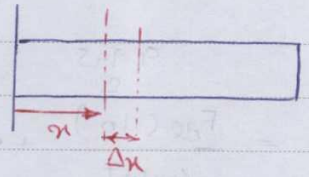
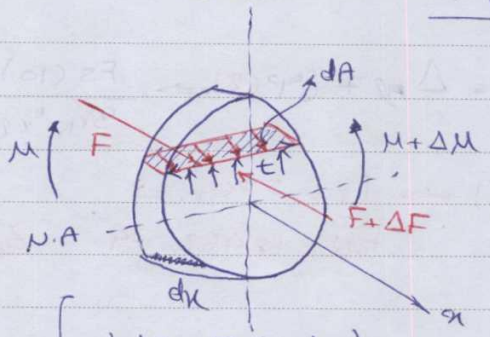
* فصل ۵ *



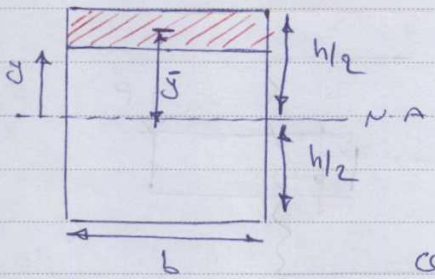


$$\sum F_x = 0 \Rightarrow \int_{A'} \sigma' dA - \int_{A'} \sigma dA - \tau (t dx) = 0$$

$$\Rightarrow \int_{A'} \left(\frac{M + dM}{I} \right) y dA - \int_{A'} \frac{M}{I} y dA - \tau t dx = 0$$

$$\Rightarrow \frac{dM}{I} \int_{A'} y dA = \tau t dx \Rightarrow \tau = \frac{1}{It} \left(\frac{dM}{dx} \right) \int_{A'} y dA$$

$$\Rightarrow \tau = \frac{VQ}{It} \; ; \; Q = \int_{A'} y dA \; ; \; V = \frac{dM}{dx}$$



$$\tau = \frac{VQ}{It} \qquad I = \frac{1}{12} b h^3$$

$$t = b \qquad V = V$$

$$Q = \underbrace{\left[y + \frac{1}{2} \left(\frac{h}{2} - y \right) \right]}_{\bar{y}} \underbrace{\left(\frac{h}{2} - y \right) b}_{A}$$

$$\Rightarrow Q = \frac{1}{2} \left(\frac{h^2}{4} - y^2 \right) b$$

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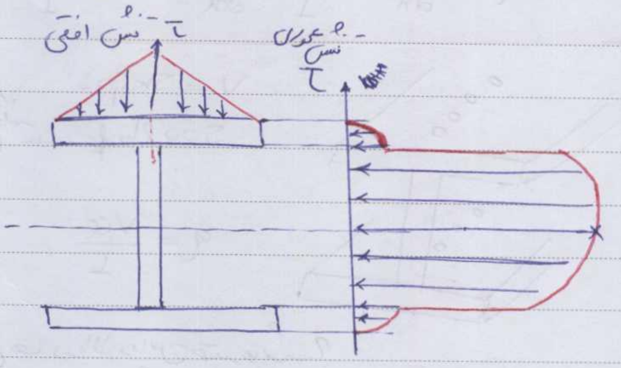
$$\tau = \frac{V \frac{1}{2}\left(\frac{h^2}{4} - y^2\right) b}{\frac{1}{12} b h^3 b} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

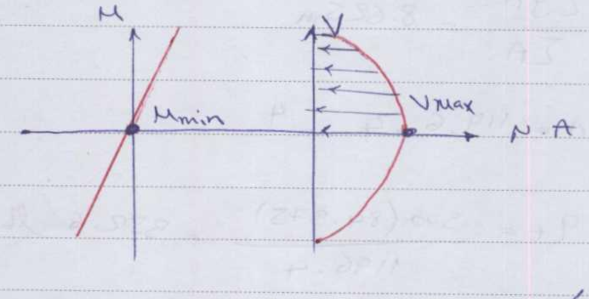
$y = h/2 \Longrightarrow \tau = 0 \qquad y = 0 \Longrightarrow \tau_{max} = \frac{6V}{4bh} = 1.5\,\frac{V}{A}$

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

$$\int_{-h/2}^{h/2} \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)\underbrace{b\,dy}_{dA} = \frac{6V}{h^3}\left[\frac{h^2}{4}y - \frac{y^3}{3}\right]_{-h/2}^{h/2} = V$$



X Section مستطیلی



* تنش برشی ماکزیمم همیشه در سطح خنثی اتفاق می‌افتد *

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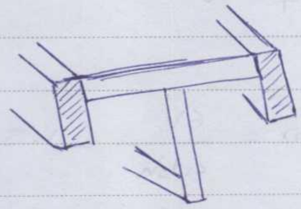
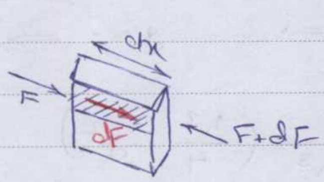
$q = \frac{dF}{dx}$ جریان برشی : نیروی برشی به ازای واحد طول



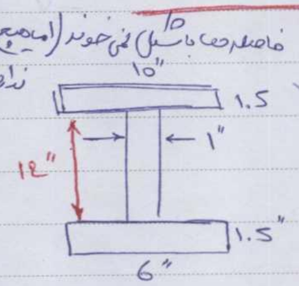


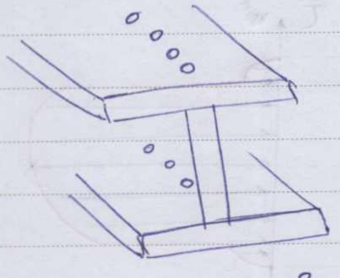
$$q = \frac{VQ}{I}$$

$$dF = \sigma' \int_A dA = \frac{dM}{I}\int y\,dA = \frac{dM}{I} Q$$

$$q = \frac{dF}{dx} = \frac{1}{I}\frac{dM}{dx} Q = \frac{VQ}{I}$$

فاصله‌ها با شکل نمی‌خوانند (ابعاد شکل اشکال دارد...!)





مثال : $V = 5 kips$

نیرویی که هر پیچ تحمل می‌کند $500\ lb_f$

$$q = \frac{VQ}{I}$$

فاصله پیچ‌ها در بالا و پایین چقدر باشد؟

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = 8.625 in$$

$$I_{N.A} = 1196.4\ in^4$$

$$q_t = \frac{500(84.375)}{1196.4} = 352.6\ lb/in \Rightarrow S_t = \frac{500}{352.6} = 1.42$$

$$q_b = \frac{500(70.87)}{1196.4} = 296.19\ lb/in \Rightarrow S_b = \frac{500}{296.19} = 1.69$$

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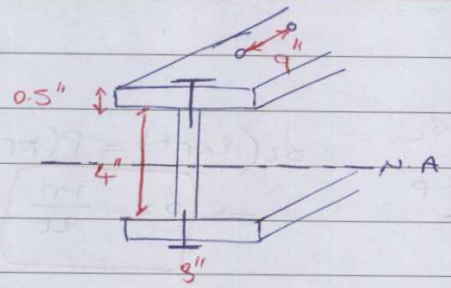
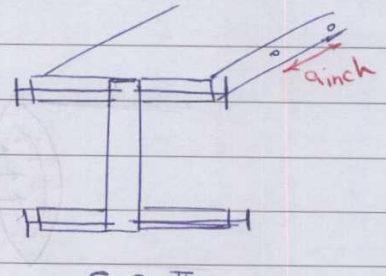
مثال:



Case I



Case II

ماکزیمم بتواند "V" را در هر دو حالت تحمل کند. قدرت هر میخ = 40 lbf

$$I = \frac{1}{12} \times 3 \times 5^3 - 2 \times \frac{1}{12} \times 1 \times 4^3 = 20.58 \text{ in}^4$$

$$\text{case I: } q = \frac{VQ}{I} \Rightarrow \frac{40}{9} = \frac{V(3 \times 0.5) \times (2.25)}{20.58} \Rightarrow V = 27.1 \text{ lb}$$

$$\text{case II: } q = \frac{VQ}{I} \Rightarrow \frac{40}{9} = \frac{V(1 \times 0.5) \times (2.25)}{20.58}$$

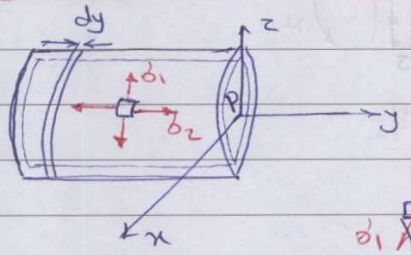
مخازن جدارنازک:

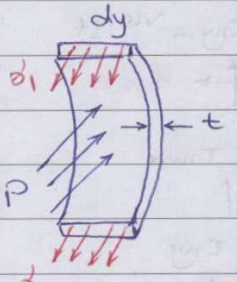
$\frac{r}{t} \geq 10$

σ_2 = تنش طولی

σ_1 = تنش محیطی

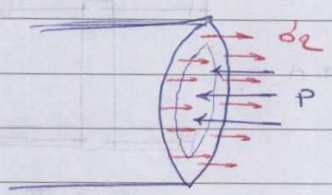
تنش "hoop"





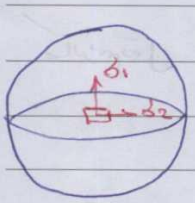
$$\Sigma F_x = 0 \Rightarrow 2(\sigma_1)(t\,dy) = P(2r\,dy)$$

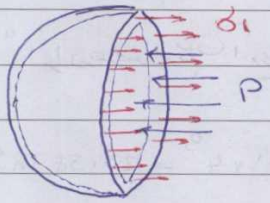
$$\Rightarrow \boxed{\sigma_1 = \frac{Pr}{t}}$$



$$\sigma_2(2\pi rt) = P(\pi r^2)$$

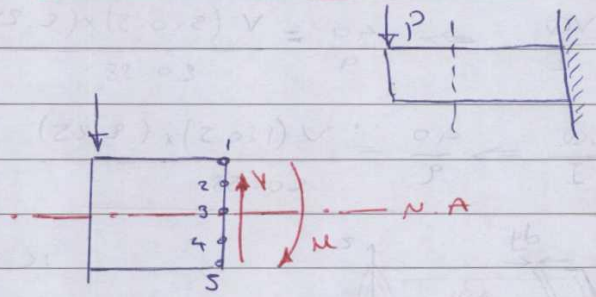
$$\Rightarrow \boxed{\sigma_2 = \frac{Pr}{2t}}$$

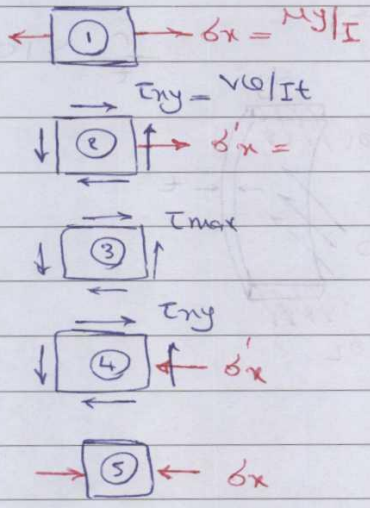




$$\sigma_1(2\pi rt) = P(\pi r^2)$$

$$\Rightarrow \boxed{\sigma_1 = \sigma_2 = \frac{Pr}{2t}}$$





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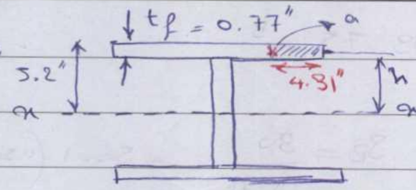


$I_{xx} = 394\ in^4$

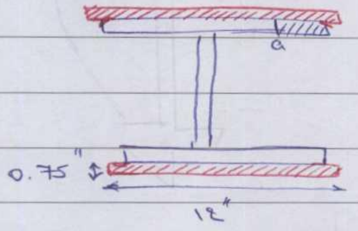
$V = 50\ kips$

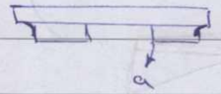
$\tau_a = \dfrac{VQ}{It}$

$h = 5.2 - \dfrac{0.77}{2} = 4.815"$

$Q = (4.31 \times 0.77) \times 4.815 = 19.98\ in^3$

$\tau = \dfrac{50}{394} \times \dfrac{19.98}{0.77} = 2.63\ ksi$





$Q = 2\big(4.31 \times 0.77 \times 4.815)$

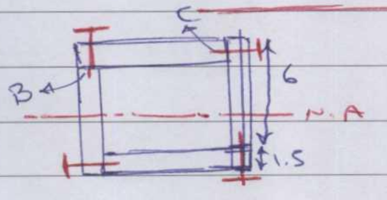
$+ (12 \times 0.75 \times 5.575\big)$

$= 82.1\ in^3$

$I = 394 + 2\left(\frac{1}{12} \times 12 \times (0.75)^3 + (12 \times 0.75)(5.575)^2\right) = 954\ in^4$

$\tau_a = \dfrac{50 \times 82.1}{954 \times (1.54)} = 2.79\ ksi$

($1.54 = 2 \times 0.77$)



قص: 30 lb   [illegible] مسموح = ?

$I_{N.A} = \frac{1}{12}(7.5)^4 - \frac{1}{12}(4.5)^4 = 229.5\ in^4$

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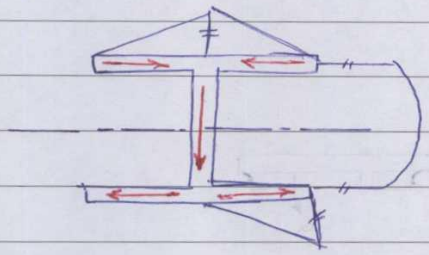
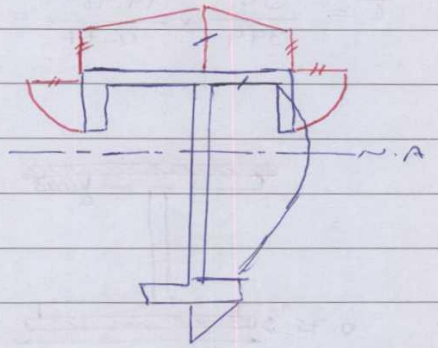
$Q_B = \left(\frac{4.5}{2} + 0.75\right)(1.5 \times 7.5) = 93.75\ in^3$

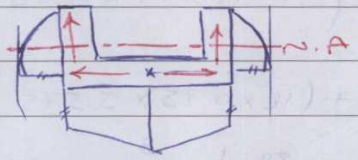
$q_B = \frac{VQ}{I} = 11.76\ lb/in$ $\qquad S_B = \frac{90}{\frac{11.76}{2}} = 50.1''\ (\text{use } 50'')$

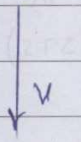
$q_c = \frac{VQ}{I} = \frac{80 \times 20.25}{229.5} = 7.059\ lb/in$

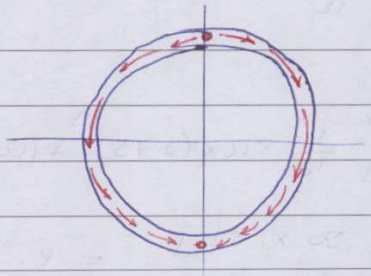
$Q_c = 3(4.5 \times 1.5) = 20.25$ $\qquad S_c = \frac{90}{(7.059)/2} = 8.5''$

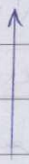


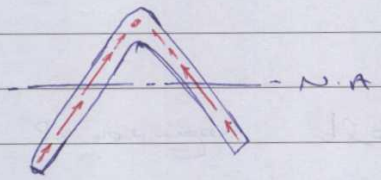












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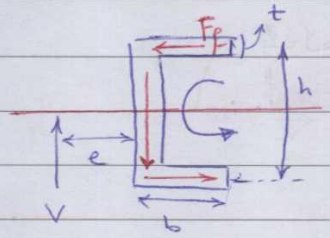
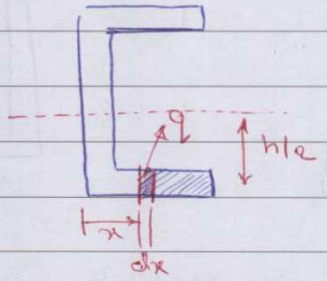


$$Ve = F_f h$$

$$F_f = \int_0^b q\,dx$$



$$q = \frac{Vc}{It}$$

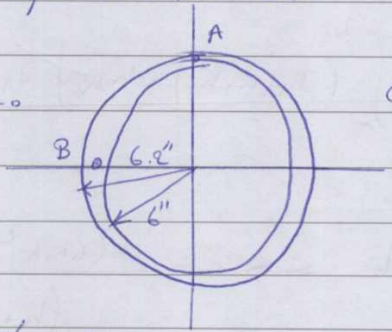
$$I = \frac{1}{12}bh^3 + 2(bt)(h/2)^2 = \frac{th^2}{2}\left(\frac{h}{6} + b\right)$$

$$q = \frac{Vc}{I} = \frac{V\frac{h}{2}(b-x)t}{\frac{th^2}{2}\left(\frac{h}{6}+b\right)} = \frac{V(b-x)}{h\left(\frac{h}{6}+b\right)}$$

$$F_q = \int_0^b q\,dx = \frac{V}{h\left(\frac{h}{6}+b\right)} \int_0^b (b-x)\,dx \Longrightarrow F_q = \frac{Vb^2}{2h\left(\frac{h}{6}+b\right)}$$

$$Ve = F_f h \longrightarrow e = \frac{b^2}{\frac{h}{3}+2b}$$

$$I = \pi/4\left(6.2^4 - 6^4\right) = 45.4\pi \text{ in}^4$$

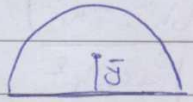


At (ends): $q_A = 0$, $I_A = 0$

$$Q_A = 0$$

$$Q_B = \sum y'A' = \frac{4(6.2)}{3\pi} \times \frac{\pi(6.2)^2}{2} - \frac{4(6)}{3\pi}\left(\frac{\pi 6^2}{2}\right)$$

$$= 14.88 \text{ in}^4$$



$$\bar{y} = \frac{4R}{3\pi}$$

$$q_B = \frac{1}{2}\frac{VQ_B}{I} = 417 \text{ lb/in}$$

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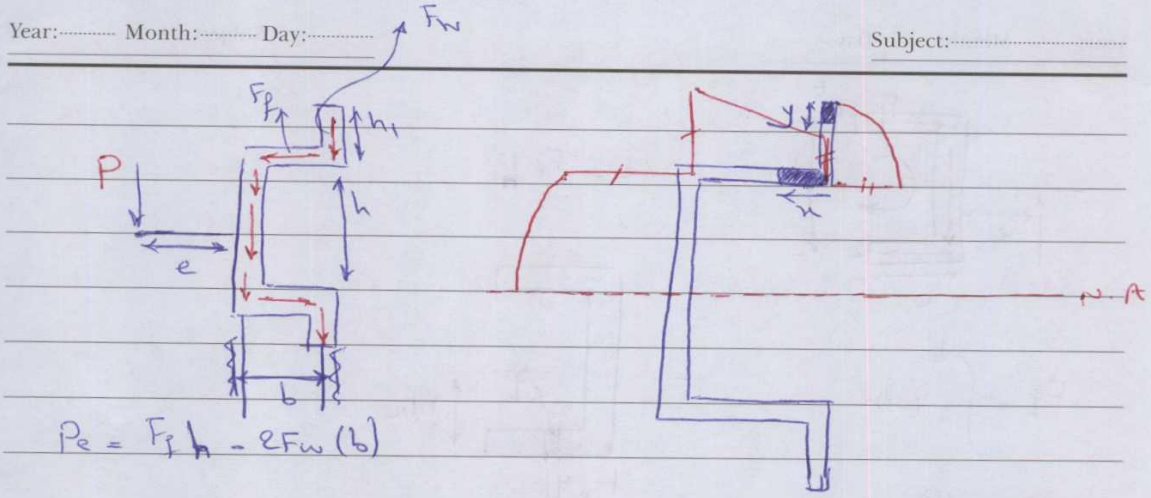

$$P_e = F_f h - e F_w (b)$$

$$I = \frac{1}{12} t(h+2h_1)^3 + 2(bt)(h_1/2)^2 = \frac{t}{12}\left((h+2h_1)^3 + 6bh^2\right)$$

$$Q_1 = \left(h_1/2 + h_1 - y/2\right) yt = t\left(h_1/2\, y + h_1 y - y^2/2\right)$$

$$Q_2 = \left(h_1/2 + h_1/2\right)(h_1 t) + h_1/2\, xt = t/2\left(h_1(h+h_1) + hx\right)$$

$$q_1 = \frac{VQ_1}{I} = Pt\left(h_1/2\, y + h_1 y - y^2/2\right)/I$$

$$q_2 = \frac{VQ_2}{I} = Pt\left(h_1(h+h_1) + hx\right)/2I$$

$$F_w = \int_0^{h_1} q_1\, dy = \frac{Pt}{I}\int_0^{h_1}(\quad)\, dy = \frac{Pt}{12I}\left(3hh_1^2 + 4h_1^3\right)$$

$$F_f = \int_0^{b} q_2\, dx = \frac{Pt}{2I}\int_0^{b}\left(h_1(h+h_1) + hx\right) dx = \frac{Pt}{2I}\left(2bh_1(h+h_1) - b^2h\right)$$

$$P_e = F_f h - e F_w b \longrightarrow e = \frac{b(6h_1h^2 + 3bh^2 - 8h_1^3)}{(h+2h_1)^3 + 6bh^2}$$

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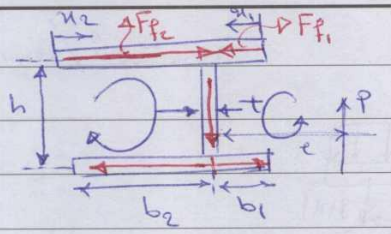
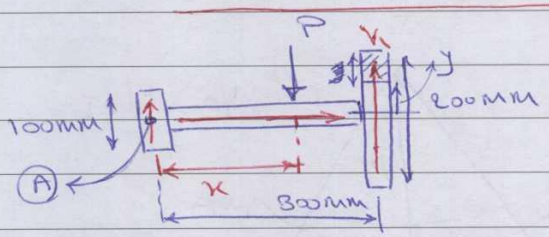
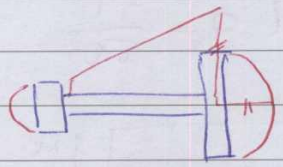


$$P_e = (F_{f_2})h - (F_{f_1})h$$

$$I = \frac{th^2}{12}\left(h + 6(b_1 + b_2)\right)$$

$$Q_1 = \frac{bI}{2} x_1 \qquad Q_2 = \frac{bI}{2} x_2 \qquad e = \frac{3(b_2^2 - b_1^2)}{h + 6(h_1 + h_2)}$$





$$\sum M_A \circlearrowleft + \cdots \Rightarrow P_e = 300 V_1$$

$$I = \tfrac{1}{12} t(0.1)^3 + \tfrac{1}{12} t(0.2)^3 = 0.75(10^{-3})\, t \ m^4$$

$$y' = y + \frac{0.1 - y}{2} = \tfrac{1}{2}(y + 0.1)$$

$$Q = y'A' = \tfrac{1}{2}(y + 0.1)(0.1 - y)\, t = \tfrac{t}{2}(0.01 - y^2)$$

$$q = \frac{VQ}{I} = 666.67\, P(0.01 - y^2)$$

$$V_1 = \int_{-0.1}^{0.1} q\, dy = 666.67\, P(0.01 - y^2)\, dy = 0.8889\, P$$

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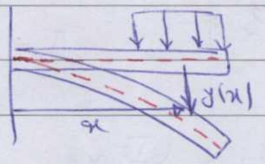
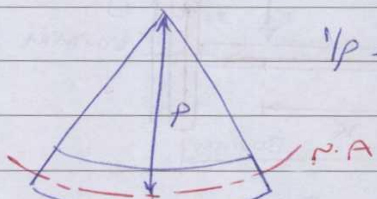
Diflection of beams / shafts — خیز تیرها / شفت‌ها

$$\begin{cases} \sigma = F/A \\ \delta l = \dfrac{FL}{AE} \end{cases}$$

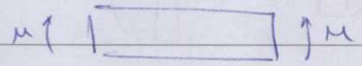


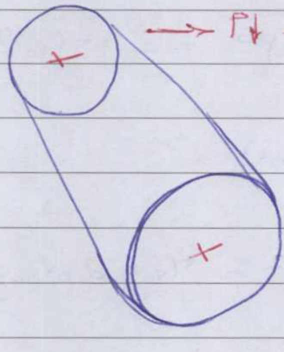
$$\begin{cases} \tau = \dfrac{T\rho}{J} \\ \phi = \dfrac{TL}{JG} \end{cases}$$

$$\sigma = \frac{My}{I}$$



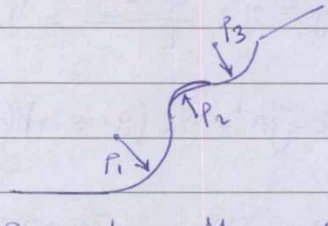
$$1/\rho = M/EI$$





→ P↓ → M↑

ممان خمشی باید وارد شود تا به دور پولی خم شود



ρ را داریم $\Rightarrow 1/\rho = \dfrac{M}{EI} \Rightarrow$ M را داریم

بنابراین ⇐ σ را داریم ⇐ $\sigma = \dfrac{My}{I}$

پلاستیک می‌رساند

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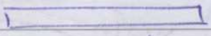
$1/\rho = \frac{M}{EI}$

مثال: تسمه اره: $0.0625''$  $0.75''$

حول یک pulley با شعاع $12''$

$\Rightarrow M = \frac{EI}{\rho}$

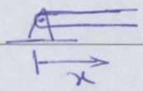
$\sigma = \frac{Mc}{I} = \frac{EIc}{I\rho} = \frac{cE}{\rho} = 29\times10^3 \times \frac{0.0625}{2\times12} = ۷۵,۵\ ksi$

$EI =$ ثابت ← در طول تیر

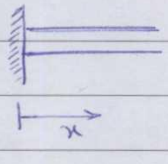
$EI \frac{d^4y}{dx^4} = -\omega(x)$

$\xrightarrow{\text{بی انتگرال}} EI \frac{d^3y}{dx^3} = V(x) \xrightarrow{\text{بی انتگرال}} \boxed{EI \frac{d^2y}{dx^2} = M(x)}$ دقت: فقط یادت باشد

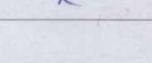
انتگرال نمی‌خواهد (تیر هم که در دست دارد)



شرایط مرزی: $y(x=0) = 0$ و $\frac{d^2y}{dx^2} = M(x=0) = 0$



شرایط مرزی: $y(x=0) = 0$ و $\frac{dy}{dx}(x=0) = 0$



$M(x=0)$

$y''(x=0) = 0$

$y'''(x=0) = 0$

$V(x=0)$

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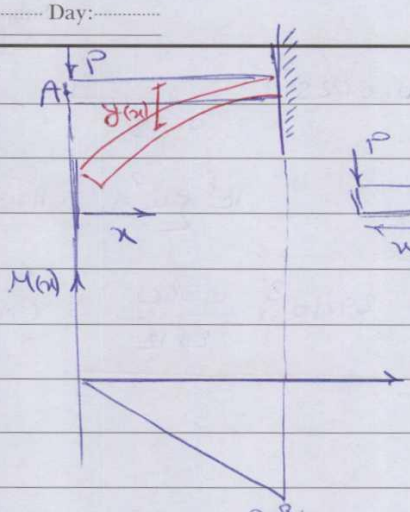
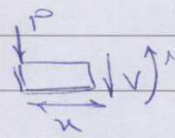
مثال:





$M(x) = -Px$

$EIy'' = M(x) = -Px$

$EIy' = -Px^2/2 + C_1$

$EIy = -Px^3/6 + C_1x + C_2$

$y(x=l) = 0 \Rightarrow C_2 = -Pl^3/3$

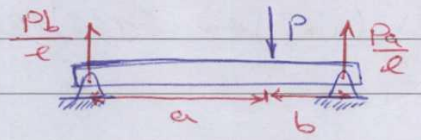
$y'(x=l) = 0 \Rightarrow C_1 = Pl^2/2$

$\Rightarrow y = \frac{P}{6EI}\left(-x^3 + 3l^2x - 2l^3\right)$

$y_{max} = y(x=0) = \frac{-Pl^3}{3EI}$ ← سوال ← most imp

$y'(x=0) = \frac{Pl^2}{2EI}$ $\quad \theta_A = \tan^{-1}\left(y'(x=0)\right)$

مثال:



$EIy'' = M(x)$

$0 < x_1 < a \Rightarrow EIy''(x_1) = M_1(x_1)$ $\quad M_1(x_1) = \frac{Pb}{l}x_1$

$EIy_1'' = \frac{Pb}{l}x_1 \Rightarrow EIy_1 = \frac{Pb}{6l}x_1^3 + C_1x_1 + C_2$

$y_1(x_1 = 0) = 0$

$a \leq x_2 \leq l \qquad EIy_2'' = M_2(x_2) \qquad M_2 = \frac{Pb}{l}x_2 - P(x_2 - a)$

$\xrightarrow{b=(l-a)} M_2 = Pa(1 - x_2/l)$

$EIy_2'' = Pa(1 - x_2/l) \longrightarrow EIy_2' = Pa\left(x_2 - \frac{x_2^2}{2l}\right) + c_3$

$\Longrightarrow EIy_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^3}{6l}\right) + c_3x_2 + c_4$

$y_2(x_2 = l) = 0$

$y_1(x_1 = a) = y_2(x_2 = a) \qquad c_1 = \frac{-Pb}{6l}(l^2 - b^2) \quad c_2 = 0$

$y_1'(x_1 = a) = y_2'(x_2 = a)$

$c_3 = \frac{-Pa}{6l}(2l^2 + a^2) \quad c_4 = Pa^3/6$

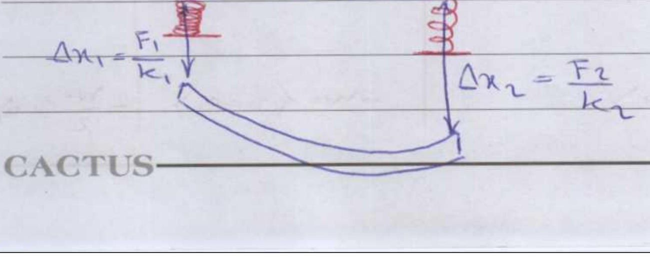
$\Longrightarrow y_1 = \frac{Pb}{6EIl}\left(x_1^3 - (l^2 - b^2)x_1\right)$

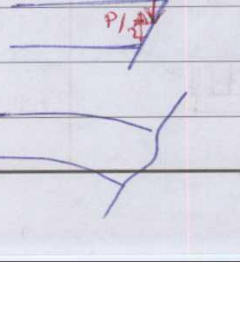
$y_2 = \frac{Pa}{6EIl}\left(3x_2^2l - x_2^3 - (2l^2 + a^2) + a^2l\right)$

برای یافتن MAX خیز باید معادلات $y_1' = 0$ و $y_2' = 0$ را حل کرد

$y_2' = 0$: جواب باید $\downarrow$ $a < x_2 < l$

$y_1' = 0$: جواب باید $\rightarrow 0 < x_1 < a$





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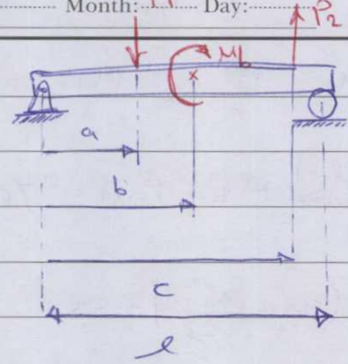


* برای تیر بالا * $0 \leq x \leq a$ $\quad M = R_1 x$

$M = R_1 x - P_1(x-a) \qquad a \leq x \leq b$

$M = R_1 x - P_1(x-a) + M_b \qquad b \leq x \leq c$

$M = R_1 x - P_1(x-a) + M_b + P_2(x-c)$

$c \leq x \leq l$

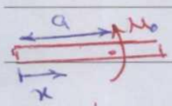
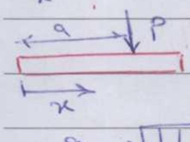
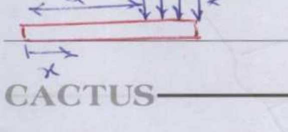
$$\langle x-a \rangle^n = \begin{cases} 0 & 0 < x < a \\ (x-a)^n & a < x < \infty \end{cases} \qquad n \geq 0$$

$$\langle x-a \rangle^n = \begin{cases} 0 & x < a \\ (x-a)^n & x \geq a \end{cases} \qquad n \geq 0$$

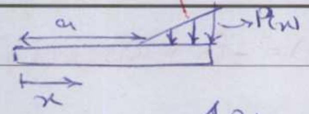
$$\int \langle x-a \rangle^n \, dx = \frac{\langle x-a \rangle^{n+1}}{n+1} + C \qquad n \geq 0$$

$$\int \langle x-a \rangle^n \, dx = \langle x-a \rangle^{n+1} \qquad n = -1, -2$$

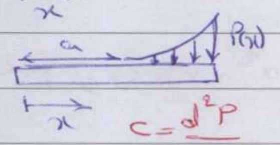
$\langle x-a \rangle^{-2}$ و $\langle x-a \rangle^{-1}$ همه جا صفرند بجز در $x = a$ بی‌نهایت می‌گردند.

Loading	Loading function $w(x)$	Shear $V = -\int w(x)dx$	$M = \int V dx$
	$M_0 \langle x-a \rangle^{-2}$	$-M_0 \langle x-a \rangle^{-1}$	$-M_0 \langle x-a \rangle^{0}$
	$P \langle x-a \rangle^{-1}$	$-P \langle x-a \rangle^{0}$	$-P \langle x-a \rangle^{1}$
	$w_0 \langle x-a \rangle^{0}$	$-w_0 \langle x-a \rangle^{1}$	$-\frac{w_0}{2} \langle x-a \rangle^{2}$

شیب خط $m = \frac{dP}{dx}$



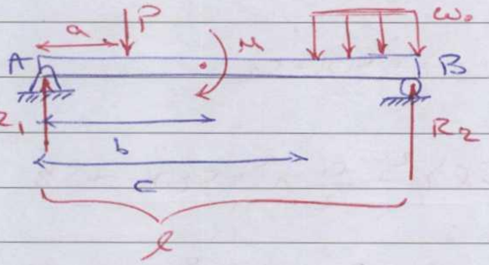
$m\langle x-a\rangle^{1}$ $\quad -\frac{m}{2}\langle x-a\rangle^{2}$ $\quad -\frac{m}{6}\langle x-a\rangle^{3}$



$\frac{c}{2}\langle x-a\rangle^{2}$ $\quad -\frac{c}{6}\langle x-a\rangle^{3}$ $\quad -\frac{c}{24}\langle x-a\rangle^{4}$

$c=\frac{d^2P}{dx^2}$

مثال:

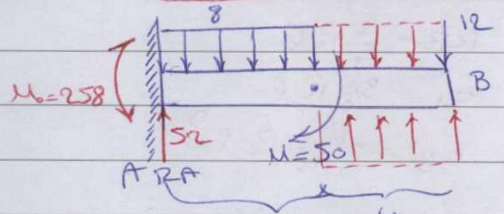


$$w(x) = -R_1\langle x-0\rangle^{-1} + P\langle x-a\rangle^{-1} - M_0\langle x-b\rangle^{-2} + w_0\langle x-c\rangle^{0}$$

اگر عکس العمل R_1 را وارد w(x) نکنیم (ما تسبالا) دو ثابت اول انتگرال صفر است.

ثابت اول برابر R_1 ، ثابت دوم برابر M_A خواهند شد

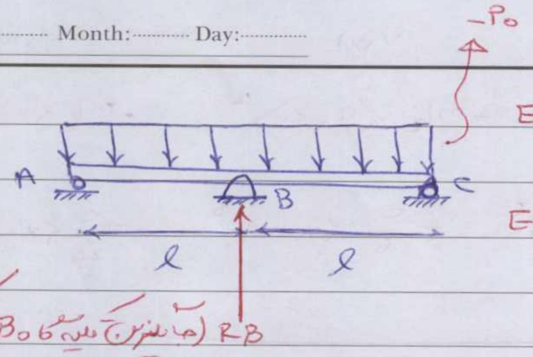
مثال:



$$w(x) = \Big(-52\langle x-0\rangle^{-1} + 258\langle x-0\rangle^{-2} + 8\langle x-0\rangle^{0}$$

$$-8\langle x-5\rangle^{0} - 50\langle x-5\rangle^{-2}\Big) = EI\,y^{(4)}_{(x)}$$

* هم RA و هم M_A را در w(x) وارد کردیم پس دیگر دو ثابت اول انتگرال را نخواهیم داشت.



(جانشین تکیه گاه B را می کنیم) RB

$$EIy^{(4)} = \omega(x) = -P_0 + R_B\langle x-\ell\rangle^{-1}$$

$$EIy^{(3)} = -P_0 x + R_B\langle x-\ell\rangle^{0} + c_1$$

$$EIy'' = \frac{-P_0}{2}x^2 + R_B\langle x-\ell\rangle + c_1 x + c_2$$

$$EIy' = -\frac{P_0}{6}x^3 + \frac{R_B}{2}\langle x-\ell\rangle^2 + \frac{c_1}{2}x^2 + c_2 x + c_3$$

$$EIy = \frac{-P_0}{24}x^4 + \frac{R_B}{6}\langle x-\ell\rangle^3 + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3 x + c_4$$

۵ مجهول c_1, c_2, c_3, c_4, R_B که با شرط مرزی و شرط لنگر:

$$y(0) = y(\ell) = y(2\ell) = 0 \qquad c_4 = c_2 = 0$$

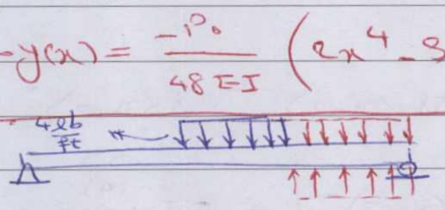
$$EIy''(0) = EIy''(2\ell) = 0 \qquad R_B = \frac{5}{4}P_0\ell$$

$$c_1 = 3/8\, P_0\ell$$

$$c_3 = -1/48\, P_0\ell^3$$

$$\Rightarrow y(x) = \frac{-P_0}{48EI}\left(2x^4 - 3\ell x^3 - 10\ell\langle x-\ell\rangle^3 + \ell^3 x\right)$$

مثال :



$$EIy^{(4)} = -4\langle x-4\rangle^0 + 4\langle x-6\rangle^0$$

$$EIy^{(3)} = -4\langle x-4\rangle^1 + 4\langle x-6\rangle^1 + c_1$$

$EIy'' = -2\langle x-4\rangle^2 + 2\langle x-6\rangle^2 + c_1x + c_2$

$y''(0)=0 \Rightarrow c_1 = 3$

$y''(8)=0 \quad c_2 = 0$

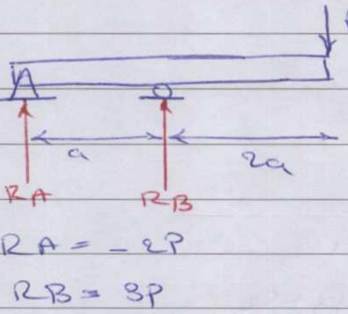
$\Rightarrow EIy'' = -2\langle x-4\rangle^2 + 2\langle x-6\rangle^2 + 3x$

$\Rightarrow EIy' = -2/3\langle x-4\rangle^3 + 2/3\langle x-6\rangle^3 + 3/2\, x^2 + c_3$

$\Rightarrow EIy = -1/6\langle x-4\rangle^4 + 1/6\langle x-6\rangle^4 + \frac{1}{2}x^3 + c_3x + c_4$

$y(0) = y(l) = 0 \Rightarrow c_4 = 0,\ c_3 = -27$

مثال:



$EIy'' = -2Px + 3P\langle x-a\rangle^1$

$EIy' = -Px^2 + 3P/2\,\langle x-a\rangle^2 + c_3$

$EIy = \frac{-Px^3}{3} + \frac{P}{2}\langle x-a\rangle^3 + c_3x + c_4$

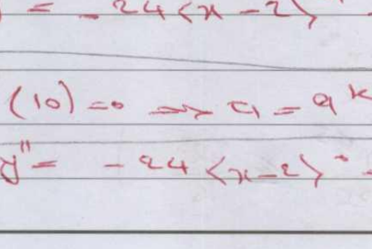
$y(0) = y(a) = 0 \Rightarrow c_4 = 0$

$c_3 = Pa^2/3$

$RA = -2P$

$RB = 3P$

مثال:



dP/dx

$EIy'''' = -24\langle x-2\rangle^{-2} - 3\langle x-4\rangle^0 + 3/6\langle x-4\rangle^1$

$EIy''' = -24\langle x-2\rangle^{-1} - 3\langle x-4\rangle^1 + \frac{3}{12}\langle x-4\rangle^2 + c_1$

$EIy'''(10) = 0 \Rightarrow c_1 = 9^k$!!! مثل پیش آمده

$EIy'' = -24\langle x-2\rangle^0 - 3/2\,\langle x-4\rangle^2 + 1/12\langle x-4\rangle^3 + 9x + c_2$

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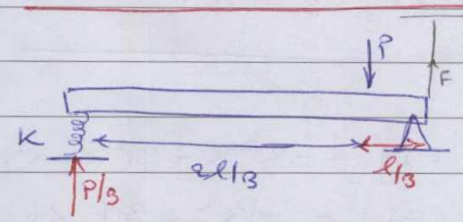
$EI\,y''(0) = 0 \Rightarrow C_2 = -80k$

$EI\theta(x) = -24\langle x-2\rangle^1 - 1/2\langle x-4\rangle^3 + \frac{1}{48}\langle x-4\rangle^4 + 9/2\,x^2 + 80x + c_3$

$\theta(6) = 0 \Rightarrow c_3 = 0$

$EIy = -12\langle x-2\rangle^2 - \frac{1}{8}\langle x-4\rangle^4 + \frac{1}{240}\langle x-4\rangle^5 + 3/2\,x^3 - 15x^2 + c_4$

$y(0) = 0 \Rightarrow c_4 = 0$



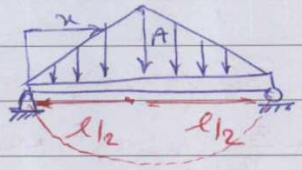
$\delta = \frac{Fl}{AE} = y(l)$

$EIy'' = Px/3 - P\langle x-2l/3\rangle^1$

$EIy' = Px^2/6 - P/2\langle x-2l/3\rangle^2 + c_1$

$EIy = \frac{Px^3}{18} - P/6\langle x-2l/3\rangle^3 + c_1x + c_2$

$\begin{cases} y(0) = -\frac{P}{3k} \\ y(l) = 0 \end{cases} \longrightarrow \begin{cases} c_2 = -\frac{PEI}{3k} \\ c_1 = \frac{PEI}{3kl} - \frac{4Pl^2}{81} \end{cases}$



$0 < x < l/2 \qquad EIy'' = Rx - \frac{Ax^3}{3l}$ ($R = \frac{Al}{4}$)

$EIy = \frac{A}{12EI}\left(l/2\,x^3 - \frac{x^5}{5l} + \frac{5l^3}{16}x\right)$

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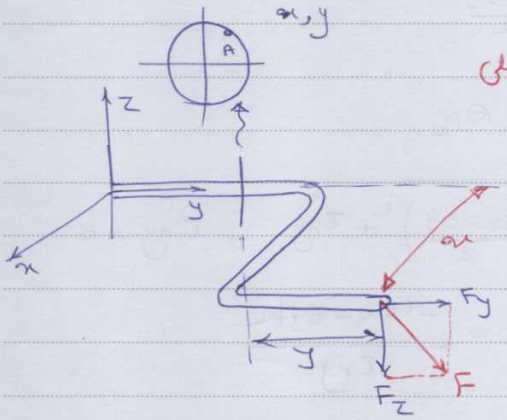
Stress Transformation. (تبدیل تنش)

$$\sigma_A = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} = \begin{pmatrix} \sigma_1^{*} & 0 \\ 0 & \sigma_2^{*} \end{pmatrix} = \begin{pmatrix} \frac{\sigma_x+\sigma_y}{2} & \tau_{max} \\ \tau_{max} & \frac{\sigma_x+\sigma_y}{2} \end{pmatrix}$$

σ_x, σ_y — σ_x, σ_y اصلی — $\bar{\sigma}, \bar{\tau}$

* تنش های اصلی

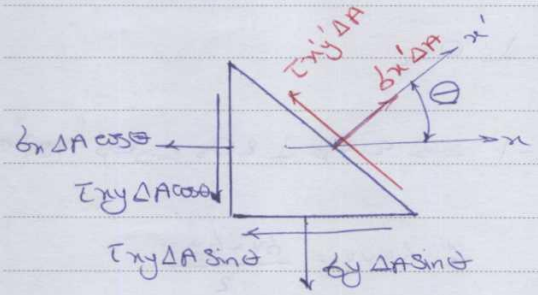


$F_z(x)$ → نیروی عمود

$F_z(y)$ → عمود ← ممان عمود

F_y → عمود

F_z → برش عمود



$$\sum F_{x'} = 0 \Rightarrow \sigma_{x'}\Delta A - \tau_{xy}\Delta A \sin\theta\cos\theta + \cdots$$

$$\cdots - \sigma_y \Delta A \sin^2\theta - \tau_{xy}\Delta A \cos\theta\sin\theta = 0$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

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$$\sum F_{y'} = 0 \Rightarrow \tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\theta \rightarrow \theta + \pi/2 \qquad \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\frac{d\gamma_{ax'}}{d\theta} = 0 \Rightarrow \tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$2\theta_{P_1}$ "(180)" $2\theta_{P_2} \Rightarrow \theta_{P_1} \perp \theta_{P_2}$

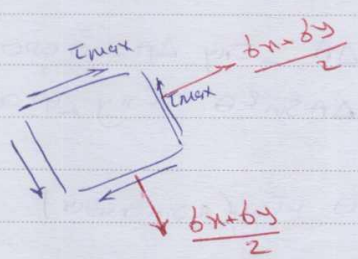
$$\sigma^{(1),(2)} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad ; \tau_{xy} = 0$$

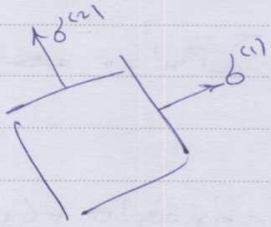
$$\frac{d\tau_{x'y'}}{d\theta} = 0 \Rightarrow \tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$2\theta_{S_1}$ "(180)" $2\theta_{S_2} \Rightarrow \theta_{S_1} \perp \theta_{S_2}$

$$(\tan 2\theta_p) * (\tan 2\theta_s) = -1 \Rightarrow 2\theta_p \perp 2\theta_s \Rightarrow \theta_p \text{ "(45)" } \theta_s$$

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad ; \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$





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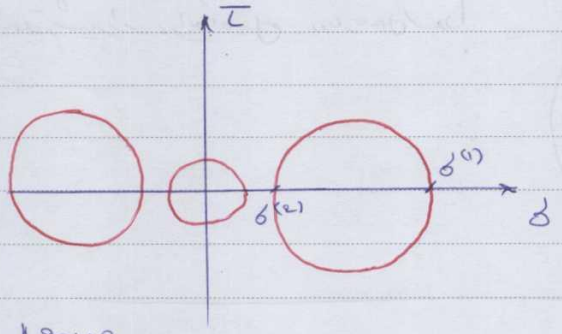
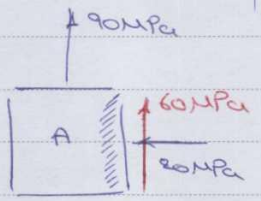
* دایره آنای مور *



مثال:



$\sigma_x = -20$

$\sigma_y = 90$

$\tau = 60$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} \longrightarrow 2\theta_{p_2} = -47.49$$

$$\Rightarrow \theta_{p_2} = -23.7$$

$$\Rightarrow \theta_{p_1} = 66.3$$

$$\sigma^{(1),(2)} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \begin{cases} \sigma^{(1)} = 116\,MPa \\ \sigma^{(2)} = -46.4\,MPa \end{cases} \xrightarrow{\text{جهت}} \text{صفحات اصلی } \theta \text{ است } (\theta_{p_1} \text{ و } \theta_{p_2})$$

$$\tan 2\theta_s = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}} \qquad 2\theta_s = 42.5$$

$$\theta_{s_1} = 21.3$$

$$\theta_{s_2} = 111.3$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 81.4$$

$$\sigma = \begin{pmatrix} -20 & 60 \\ 60 & 90 \end{pmatrix} = \begin{pmatrix} 116 & 0 \\ 0 & -46.4 \end{pmatrix} = \begin{pmatrix} 35 & 81.4 \\ 81.4 & 35 \end{pmatrix}$$

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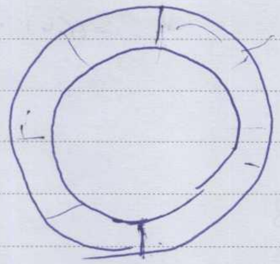
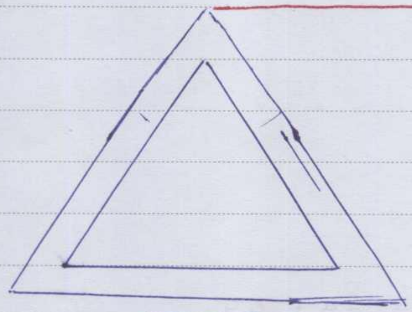


* حلقه کامل، حلقه شیاردار، دایره کامل دوباره حل شود!
مثلث هم حل شود



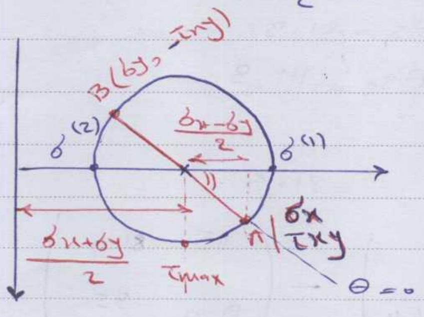
$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right) = \left(\frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)$$

$$\left(\tau_{x'y'}\right) = \left(-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)$$

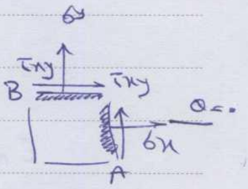
$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

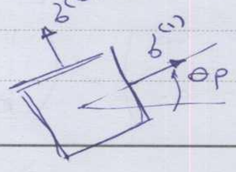
$$(x - a)^2 + (y-b)^2 = R^2$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}}$$





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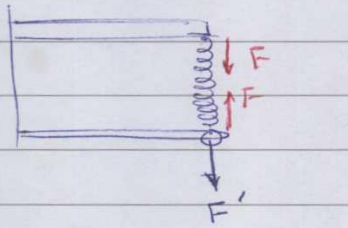
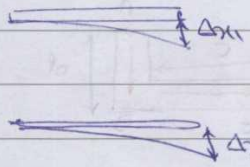
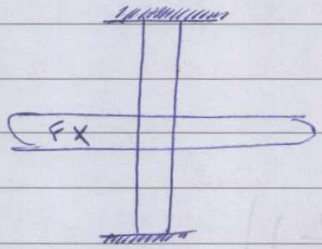
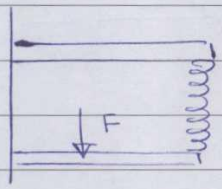
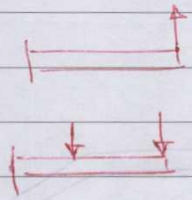


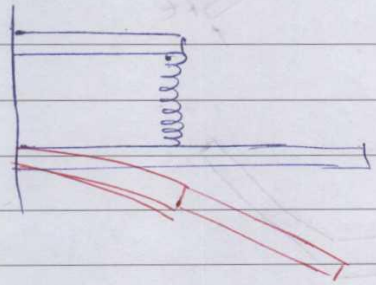


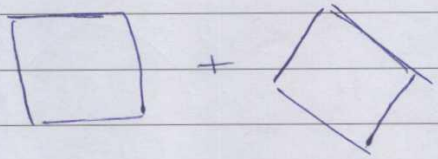
$|\Delta x_2 - \Delta x_1| = F/k$

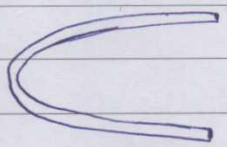


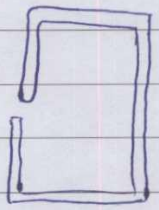












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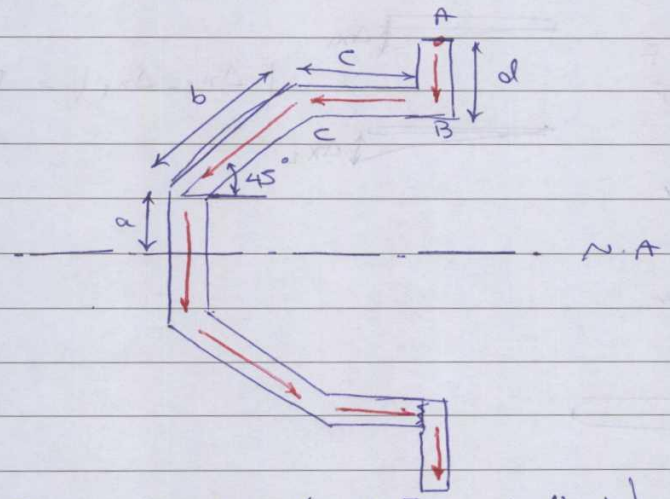
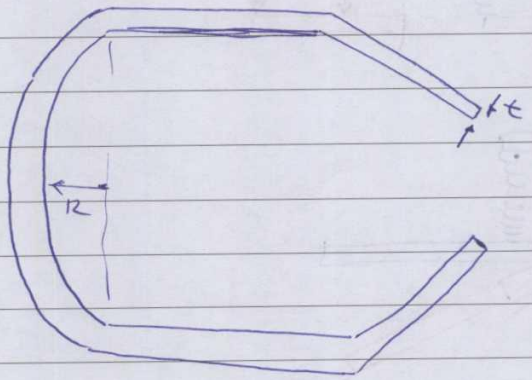


$$A-B:\ \tau_{xy} = \frac{V}{It}\left(Ut\left(a + b\sqrt{2}/2 + d - \frac{U}{2}\right)\right)$$

B-C ; τ_{xz} =



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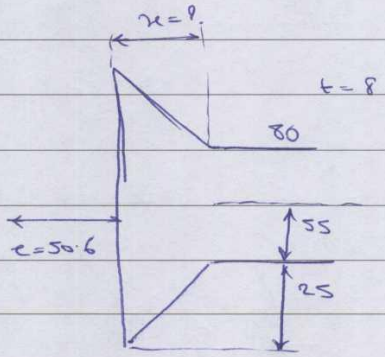
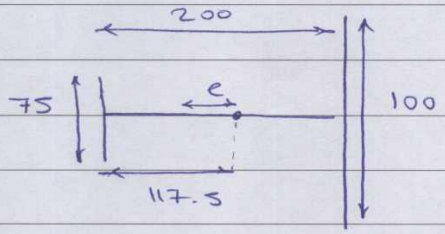


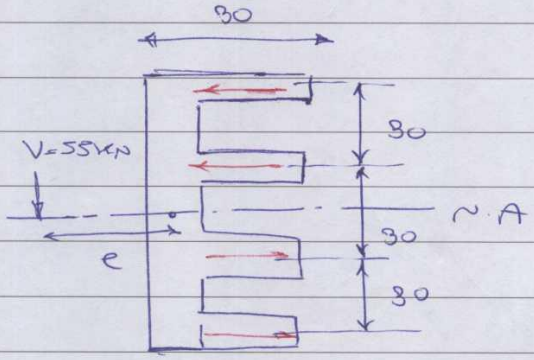
t=6



$I_y = 1.902 \times 10^6$

$I_z = 1.526 \times 10^6$

(e=81.6 maybe)



$I_z = 0.9333 \times 10^6$

$e = 9.12$